الآية

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صدق الله العظيم

سورة العلق
Dedication

This thesis is dedicated to:

Great teacher and messenger, Mohammed (May Allah bless him and grant him), who taught us the purpose of life.

Sudan University of Science and Technology, my second magnificent home.

My Mother dear, fountain of patience and optimism and hope,

My Father, who leads me through the valley of darkness with light of hope and support,

Beloved brother and sisters; particularly my dearest brother Awed, who stands by me when things look bleak,

Who paved me way of science and knowledge, Mona Siddig.

My love, whom I can't force myself to stop loving.

Friends who encourage and support me.
Aknowledgments

Thanks and appreciation to the one who his knowledge lit my mind and who taught me how to learn,

Dr. Mohamed Hassan Mohamed Khabir.
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Abstract (English)

In this thesis we discuss the necessary background material for the description of spline-based finite element methods. We explain the basic finite element idea by constructing the classical Ritz-Galerkin scheme for Poisson’s equation. We define the concept of ellipticity and the Lax-Milgram existence theorem for variational problems of the Poisson’s equation. We introduce and define the concept of splines and B-splines. We construct the fundamental recurrence relation, which allows us to evaluate B-splines efficiently and to compute their polynomial segments. We also discuss algorithms for grid refinement and computation of scalar products for B-splines and their derivative. Then we construct the finite element bases functions in regular grids using B-splines and multivariate B-splines. Finally, we discuss the approximation of Poisson’s equation with essential and natural boundary conditions.
Abstract (Arabic)

ناقشنا في هذا البحث، الخلفية العلمية المهمة لوصف الدوال اللسنية لتكون أساساً لطرائق العنصر المنتهي. أوضحنا الفكرة الأساسية للعنصر المنتهي بإنشاء طريقة ريتز-جريجرن التقليدية لمعادلة بويسون. عرفنا مفهوم الناقصية ونظرية لاكس-ميلجرام لوجود مسائل التغابر لمعادلة بويسون. قدمنا وعرفنا مفهوم الدوال اللسنية و ب-اللسنية. أنشأنا علاقة التكرار الأساسية التي تسمح لنا بتطبيق ب-اللسنية بدقة وتحسين تقسيمات كثيرات الجرد. أيضاً ناقشنا خوارزميات تحسين التقسيمات وتحسين الضرائب القياسي ل- ب-اللسنية ومشتقاتها. ثم كوننا دوال الأساس للعنصر المنتهي في تقسيمات منتظمة باستخدام ب-اللسنية و ب-اللسنية متعددة المتغيرات. أخيراً، ناقشنا التقريب لمعادلة بويسون مع الشروط الحدية الأساسية والطبيعية.