



Sudan University of Science and Technology

College of Graduate studies

**Some aspects of the Higgs Boson in the
Standard Model**

بعض خواص جسيم هيغز في النموذج العياري

**A thesis submitted as partial fulfillment of the
requirements for the degree of M. Sc. in physics**

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الآية

قال تعالى :

اللَّهُ نُورُ السَّمَاوَاتِ وَالْأَرْضِ ۚ مِثْلُ نُوْرِهِ كَمِشْكَاةٍ فِيهَا مِصْبَاحٌ ۚ الْمِصْبَاحُ فِي زُجَاجَةٍ ۚ الزُّجَاجَةُ كَأَنَّهَا كَوْكَبٌ دُرِّيٌّ يُوقَدُ مِنْ شَجَرَةٍ مُبَارَكَةٍ زَيْتُونَةٍ لَا شَرْقِيَّةٍ وَلَا غَرْبِيَّةٍ يَكَادُ زَيْتُهَا يُضِيءُ وَلَوْ لَمْ تَمْسَسْهُ نَارٌ ۚ نُورٌ عَلَى نُورٍ ۚ يَهْدِي اللَّهُ لِنُورِهِ مَنْ يَشَاءُ ۚ وَيَضْرِبُ اللَّهُ الْأَمْثَالَ لِلنَّاسِ ۚ وَاللَّهُ بِكُلِّ شَيْءٍ عَلِيمٌ

صدق الله العظيم

سورة النور: الآية 35

Dedication

To my mother for her unlimited love.

To my father for his constructive advice throughout my live.

To my brothers and my sister.

To all of my friends.

Hassan

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After thanking Allah, I would like to thank to my supervisor of this thesis, **Dr. Rawia Abdelgani Elobaid** for the valuable guidance and advice.

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Abstract

In this thesis was studied some aspect of the Higgs boson in the Standard Model of Particle Physics. Also was calculated the Higgs boson as function of its mass as well as the branching ratios from $m_h = 100 \text{ GeV}$ to $m_h = 400 \text{ GeV}$. Furthermore was calculated the Higgs decay rates into W and Z gauge bosons and f fermions in the Standard Model of Particle Physics. We find that in order for the Higgs to decay to any particle in this model, the mass of the Higgs should be $m_h \geq 2m_{W,Z,f} \text{ GeV}$. Also we found that, the total Higgs decay rate and its branching ratios increase as the Higgs mass increases.

ملخص البحث

في هذا البحث درست بعض خواص جسيم هيغز بوزون في النموذج القياسي (العياري) للجسيمات الاولية. وايضا تم حساب معدل التحلل لجسيم هيغز و حسبت نسبة معدل التحلل لجسيم هيغز الى التحلل الكلي لجسيم هيغز حيث تم استخدام قيم لكتلة هيغز من (100 – 400) جيجا الكترون فولت. إضافة الي ذلك حسبت معدل تحلل هيغز الى W و Z بوزون و f فيرميون في النموذج العياري. ووجد انه لكي يتحلل جسيم هيغز الى فيرميونات أو بوزونات يجب ان تكون كتلة جسيم هيغز اكبر من أو تساوي كتلة الجسيم المتحلل له. ووجد ايضا ان معدل التحلل الكلي ونسبة التحلل الى التحلل الكلي تزيد بزيادة كتلة جسيم هيغز.

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Chapter One

Introduction

(1-1) Introduction:

In this thesis, we will discuss and explore the importance of studying the Higgs boson decay and branching ratio in the Standard Model of particle physics. The Higgs is a new particle in the particle physics to solve the mass issues in the standard model, it is really forms one of the foundations of the electroweak sector of the standard model; it is spontaneously breaks the symmetry and responsible of giving masses to both fermions and gauge bosons.

(1-2)The importance and problem of the thesis:

The main objective of the Large Hadron Collider (LHC) is to discover the Higgs boson but also the explore physics at TeV scale. This scalar particle has been discovered in 2012 and still there is some confusion whether we have discovered really the Higgs boson or not, it could be other scalar particle. As such more work on the properties of the Higgs boson and its couplings need to be reconsidered. Therefore, we will study the decay rate and its branching ratio for the Higgs boson to predict the probability of finding the Higgs boson and which decay channel to look for.

(1-3)The main objectives of the thesis:

This thesis is meant to explain what is higgs boson ,and what is standard model. It is also to discuss how the higgs field interacts with other particels and gives them mass and then calculate the decay rates and branching ratios for the Higgs in the Standard Model.

(1-4) Literature review:

- F. Englert and R. Brout (Physical Review Letter) in 1964

Broken symmetry and the mass of gauge vector meson:

It is of interest to inquire whether gauge vector mesons acquire mass through interaction; by a gauge vector meson we mean a Yang-Mills field associated with the extension of a Lie group from global to local symmetry.

The importance of this problem resides in the possibility that strong-interaction physics originates from massive gauge fields related to a system of conserved currents. In this note, we shall show that in certain cases vector mesons do indeed acquire mass when the vacuum is degenerate with respect to a compact Lie group. Theories with degenerate vacuum (broken symmetry) have been the subject of intensive study since their inception by Nambu. A characteristic feature of such theories is the possible existence of zero-mass bosons which tend to restore the symmetry. We shall show that it is precisely these singularities which maintain the gauge invariance of the theory, despite the fact that the vector meson acquires mass.

➤ Peter .W. Higgs (Physical Review Letter) in 1964

Broken symmetries and the mass of gauge boson:

Was shown that the Goldstone theorem, that Lorentz covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson has drawn attention: that the scalar zero-mass excitations of a superconducting neutral

Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

(1-5)The outline of the thesis:

This thesis is structured as follow: In chapter one, we give mini introduction about thesis, in chapter two, we give brief introduction to the standard model of particle physics, in chapter three we discussed the Higgs mechanism, in chapter four we introduce the calculations of Higgs decay rates and the branching ratio, also we present in chapter four our numerical results and discussion, finally in chapter five we discuss and conclude our results.

Chapter Two

Introduction to the Standard Model

(2-1) Introduction:

After the discovery of the last piece of the scalar Standard Model (SM), the Higgs boson particle in 2012 at the Large Hadrons Collider (LHC). There have been many attempts to understand the behavior of this particle in term of its interaction and its couplings to other particles in many models.

Modern theory called the standard model attempts to explain all the phenomena of particle physics in term of properties and interactions of small number of particles in three distinct types. The first are called leptons and quarks, both of them have spin-1/2 named fermions. The second one is a set of spin-1 named bosons, which act as force carriers in the theory. The last one is scalar particle has spin 0, which breaks the symmetry and give masses to all particles [1].

The most familiar example of leptons is the electron, which is bound in atoms by electromagnetic interaction, one of the four fundamental force in nature. Another well-known example is neutrino, which is a light neutral particle observed in the decay products of some unstable nuclei (β^- decay). The force responsible for the β^- decay of nuclei is called the weak interaction. In addition to leptons, another class of particles called hadrons is also observed in nature. Familiar examples are the neutron and proton (collectively called nucleons). In the standard model, these are not considered to be elementary, but are made of quarks bound together by the third force of nature, the strong, interactions. In addition to the strong interaction, weak and electromagnetic interactions between quarks and leptons, there is a fourth force of nature, gravity. However the gravitational interaction between elementary particles is , in practice , so small compared with the other three interactions that we shall neglect it [1,2].

These forces are associated with elementary spin-1 (bosons), the gauge bosons or force carriers mentioned later. Consider for example the electromagnetic interaction. In classical physics, the interaction between two charged particles is transmitted by electromagnetic waves which are continuously emitted and absorbed. This is an adequate description at long distances, but at short distances the quantum nature of the interaction is transmitted discontinuously by the exchange of spin-1 photons. These are the force carriers of the electromagnetic interaction and the long – range nature of the force is related to the fact that the photons have ($m_\gamma = 10^{-64}$) is very small mass, approximately zero mass [1].

The weak and strong interactions are also associated with the exchange of spin-1 particles. For the weak interaction, they called W and Z bosons, they are very massive. The resultant force is short range, many applications effect by interaction in point. The gluon is a particle has strong interaction which is mass less like the photon [3].

(2-2) Leptons:

Leptons are one of the two classes of elementary particles of matter, the fermions, leptons are not influenced by the strong interaction. In fact leptons are twelve types in nature, but the main leptons are three types: the electron (e^-), the muon (μ^-), and the tauon (τ^-). Each one of these has an associated neutrino, the electron neutrino, the muon neutrino and the tauon neutrino. They are occurring in pairs called generation, which can be written as doublets:

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

Note that we have an important principle for particle interactions is the conservation of lepton number, every lepton pair has its own quantum number as:

$$L_e = N(e^- + \nu_e) - N(e^+ + \bar{\nu}_e) \quad (2.1)$$

$$L_{\mu} = N(\mu^{-} + \nu_{\mu}) - N(\mu^{+} + \bar{\nu}_{\mu}) \quad (2.2)$$

$$L_{\tau} = N(\tau^{-} + \nu_{\tau}) - N(\tau^{+} + \bar{\nu}_{\tau}) \quad (2.3)$$

Obviously, the total lepton number is the sum of these three

$$L = L_e + L_{\mu} + L_{\tau} \quad (2.4)$$

All known interaction conserve the total lepton number.

(2-3) Leptons properties:

(2-3-1) Electron:

As one of the leptons, the electron is viewed as one of the fundamental particles. It is a Fermion of spin- 1/2 and therefore constrained by the Pauli Exclusion Principle, a fact that has key implications for the building up of the periodic table of elements.

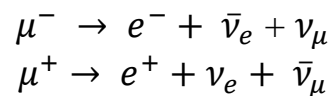
The electron's antiparticle, the positron, is identical in mass but has a positive charge. If an electron and a positron meet each other, they will annihilate with the production of two gamma-rays. On the other hand, one of the mechanisms for the interaction of radiation with matter is the pair production of an electron-positron pair. Associated with the electron is the electron neutrino [3].

Table(2-1): electrons and its quantum numbers

Particle	Symbol	Anti-particle	Rest mass MeV/c ²	L(e)	L(muon)	L(tau)	Lifetime (seconds)
Electron	e^{-}	e^{+}	0.511	+1	0	0	Stable
Neutrino (Electron)	ν_e	$\bar{\nu}_e$	$0(<7 \times 10^{-6})$	+1	0	0	Stable

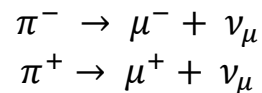
(2-3-2) Muon:

The muon is a lepton which decays to form an electron or positron.



The fact that the above decay is three-particle decay is an example of the conservation of lepton number; there must be one electron neutrino and one muon neutrino or antineutrino in the decay.

The lifetime of the muon is 2.20 microseconds. The muon is produced in the upper atmosphere by the decay of pions produced by cosmic rays:



Measuring the flux of muons of cosmic ray origin at different heights above the earth is an important time dilation experiment in relativity.

Muons make up more than half of the cosmic radiation at sea level, the remainder being mostly electrons, positrons and photons from cascade events. The average sea level muon flux is about 1 muon per square centimeter per minute.

Table(2-2): muons and its quantum numbers

Particle	Symbol	Anti-particle	Rest mass MeV/c ²	L(e)	L(muon)	L(tau)	Lifetime (seconds)
Muon	μ^-	μ^+	105.7	0	+1	0	2.20×10^{-6}
Neutrino (Muon)	ν_μ	$\bar{\nu}_\mu$	0(<0.27)	0	+1	0	Stable

(2-3-3) Tauon:

The tau is the most massive of the leptons, having a rest mass some 3490 times the mass of the electron, also a lepton. Its mass is some 17 times that of the muon, the other massive lepton.

Table(2-3): tauons and its quantum numbers

Particle	Symbol	Anti-particle	Rest mass MeV/c ²	L(e)	L(muon)	L(tau)	Lifetime (seconds)
Tau	τ^-	τ^+	1777	0	0	+1	2.96×10^{-13}
Neutrino (Tau)	ν_τ	$\bar{\nu}_\tau$	0(<31)	0	0	+1	Stable

(2-4) Quarks:

A quark is one of the fundamental particles in physics, has spin-1/2 the (fermions), they join together to form hadrons, such as protons and neutrons. There are 36 types of quarks, but the main types of quarks are six (up, down, charm, strange, top and bottom), occurring in pairs, or generations as follow:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$$

And also we have six anti-quarks are:

$$\begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}, \begin{pmatrix} \bar{c} \\ \bar{s} \end{pmatrix}, \begin{pmatrix} \bar{t} \\ \bar{b} \end{pmatrix}$$

Table(2-4): quarks properties

Quark	Symbol	Spin	Charge	Baryon number	S	t	c	b	Mass
Up	u	$1/2$	$+2/3$	$1/3$	0	0	0	0	5 MeV
Down	d	$1/2$	$-1/3$	$1/3$	0	0	0	0	10 MeV
Charm	c	$1/2$	$+2/3$	$1/3$	0	0	+1	0	1.3 GeV
strange	s	$1/2$	$-1/3$	$1/3$	-1	0	0	0	200 MeV
Top	t	$1/2$	$+2/3$	$1/3$	0	+1	0	0	175 GeV
Bottom	b	$1/2$	$-1/3$	$1/3$	0	0	0	-1	4.5 GeV

- The proposal of Gell- Mann and Zweig in 1964 was the hadrons are spin- $1/2$ fermions, called quarks. The rich spectrum of hadrons they introduced three types or flavor of quarks: up, down, and strange. The up and down quarks form an isospin doublet (for up, $I_3 = +1/2$) and (for down, $I_3 = -1/2$)
- The strange quark is an isospin singlet with strangeness = -1, but the up and down quarks have strangeness equal zero
- In three scheme the baryons are bound state of three quarks to produce baryon number (B =1).
- The study of quarks and the interaction between them through the strong force is called quantum chromo dynamics, in this field there are three colors of quarks: red (R), green (G) and blue (B). These are of course not real colors, but just away to differentiate between the quarks, but the particles states in nature are colorless states [5].

(2-5) Electroweak theory of the standard model :

(2-5-1)Electroweak theory:

We will explore the electroweak part of the standard model of particles physics, which unifies the electromagnetic and weak interactions. The gauge group that dose this is [$SU(2)$ and $U(1)$].

The weak interactions are mediated by the $SU(2)$ gauge bosons, which includes the charged W^+ , W^- and the neutral Z^0 . The $U(1)$ sector of the interaction is the electromagnetic interaction which is mediated by the mass less photon. The theory that describe the electroweak interaction is known as the Weinberg – Salam model, after the two discoverers of the theory. They shared the nobel prize with Sheldon Glashow in 1979 for the development of this theory and their prediction of the W^+ , W^- and Z^0 masses.

The Higgs field is introduce into the model causing spontaneous symmetry breaking. This leads the electron and its heavy partners, the muon (μ) and tau (τ) to acquire mass in addition. The gauge boson (W^+ , W^- and Z^0) acquire mass also, but the photon remain mass less, so far so good. The result are in good agreement will experiment.

However , the Weinberg – Salam model also predicts that the neutrinos are mass less, recent experimental evidence indicates that while their mass may be small than 1ev,neutrions probably do have mass. This problem is currently one of the great out standing problem in particle physics. And solving the neutrino mass problem may lead to new physics beyond the standard model [6].

(2-5-2)Right and left handed Spinors:

Let's briefly review concept of left and right handed spinors, we write Dirac spinor as a two component object, with top component being the right-handed spinor and the lower component being the left- handed spinor.

$$\Psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

Each component, ψ_R and ψ_L , is itself a two component object. We can pick out the left-handed and right-handed component of Dirac field Ψ by using an operator composed of the identity and r_s matrix.

$$r_s = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2.5)$$

Now let's see how we can pick out the left-handed and right-handed components of Ψ [4].

First we write:

$$\frac{1}{2}(1 - r_s) = \frac{1}{2} \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Hence

$$\frac{1}{2}(1 - r_s) \Psi = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} = \begin{pmatrix} 0 \\ \psi_L \end{pmatrix} = \psi_L$$

Similarly, we have

$$\frac{1}{2}(1 + r_s) \Psi = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} = \begin{pmatrix} \psi_R \\ 0 \end{pmatrix} = \psi_R$$

Also notice that we can write Dirac field as:

$$\Psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} = \begin{pmatrix} 0 \\ \psi_L \end{pmatrix} + \begin{pmatrix} \psi_R \\ 0 \end{pmatrix} = \psi_L + \psi_R$$

(2-5-3) A Massless Dirac Lagrangian:

We begin with the standard Dirac Lagrangian, setting the mass term to zero, this gives

$$L = i \bar{\Psi} \partial^\mu \partial_\mu \Psi \quad (2.6)$$

Where as usual

$$\bar{\Psi} = \Psi^\dagger \gamma^0$$

We wish to split up the Lagrangian into two parts, one for the left-handed spinor and one for the right-handed spinor. This actually very straight forward.

Proceeding we have:

$$\begin{aligned} L &= i \bar{\Psi} \partial^\mu \partial_\mu \Psi = (\bar{\Psi}_L + \bar{\Psi}_R) \partial^\mu \partial_\mu (\Psi_L + \Psi_R) \\ &= i \bar{\Psi} \partial^\mu \partial_\mu \Psi_L + i \bar{\Psi} \partial^\mu \partial_\mu \Psi_R \\ &\quad + i (\bar{\Psi}_L \partial^\mu \partial_\mu \Psi_R + \bar{\Psi}_R \partial^\mu \partial_\mu \Psi_L) \end{aligned} \quad (2.7)$$

The last term actually vanishes. This because Ψ

$$\begin{aligned} \bar{\Psi}_L \partial^\mu \partial_\mu \Psi_R &= \left(\frac{1-r_s}{2} \right) \bar{\Psi} \partial^\mu \partial_\mu \left(\frac{1+r_s}{2} \right) \Psi \\ &= \frac{1}{4} (1 - r_s + r_s - r_s^2) \bar{\Psi} \partial^\mu \partial_\mu \Psi \\ &= \frac{1}{4} (1 - r_s^2) \bar{\Psi} \partial^\mu \partial_\mu \Psi \end{aligned}$$

Since

$$r_s^2 = 1$$

The mixed term vanish, so we are left with

$$L = i \bar{\Psi}_L \partial^\mu \partial_\mu \Psi_L + i \bar{\Psi}_R \partial^\mu \partial_\mu \Psi_R \quad (2.8)$$

And Lagrangian separates nicely into left –handed and right-handed parts. In the case of the electroweak theory, these is an asymmetry between left –handed and right-handed weak interactions[6,9].

(2-5-4) Unitary transformations and the gauge fields of theory:

Next we consider the possible symmetries of the theory and proceed

to introduce the gauge boson as mentioned above. In electroweak theory has two independent symmetries SU(2) and U(1), we called the combination

$$SU(2) \times U(1)$$

The SU(2) leads to three gauge bosons as mentioned here

$$SU(2): W_\mu^1, W_\mu^2, W_\mu^3$$

The conservation of the weak hyper charge (Y) actually arises from invariance under U(1) transformation, hence there is additional gauge field associated with U(1) invariance, we denote this field as B_μ

So

$$U(1): B_\mu$$

After we introduce the gauge fields, the Lagrangian will be expanded as

$$L = L_{leptons} + L_{gauge} + L_{Yukawa} \quad (2.9)$$

Let's take a look at U(1) transformation first, it is clear by looking at the lagrangian that it is invariant under a standard U(1) transformation on the right-handed spinor

$$\Psi_R \rightarrow \hat{\Psi}_R = e^{iB} \Psi_R$$

Where B is scalar. This transformation does not change the Lagrangian

$$\begin{aligned} L \rightarrow \hat{L} &= i \bar{\Psi}_L \partial^\mu \partial_\mu \Psi_L + i \bar{\Psi}_R e^{-iB} \partial^\mu \partial_\mu e^{iB} \Psi_R + i \bar{L} \partial^\mu \partial_\mu \\ &= i \bar{\Psi}_L \partial^\mu \partial_\mu \Psi_L + i \bar{\Psi}_R \partial^\mu \partial_\mu \Psi_R = L \end{aligned} \quad (2.10)$$

So clearly the Lagrangian is invariant under : $(\Psi_R \rightarrow \hat{\Psi}_R = e^{iB} \Psi_R)$

Naively, one would expect the U(1) transformation to be the same for the left-handed field

$$\Psi_L \rightarrow \hat{\Psi}_L = e^{iB} \Psi_L$$

we already know that the left-handed and right-handed fields have different weak hyper charge, so we expect them to transform differently. The correct transformation for the left –handed field is of the form

$$\{ Y_R = -2 , \text{ But } Y_L = -1, \text{ the left-handed interact at half, } n = \frac{1}{2} \}$$

so, the correct transformation is

$$\Psi_L \rightarrow \hat{\Psi}_L = e^{iB} \Psi_L$$

we can arrange the neutrino, the left-handed and right-handed for the electron into a single object as

$$\begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix}$$

Then U(1) transformation can be written in nice matrix form as

$$\begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix} \rightarrow \begin{pmatrix} \nu'_e \\ e'_L \\ e'_R \end{pmatrix} = \begin{pmatrix} e^{\frac{iB}{2}} & 0 & 0 \\ 0 & e^{\frac{iB}{2}} & 0 \\ 0 & 0 & e^{\frac{iB}{2}} \end{pmatrix} \begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix}$$

Now given a gauge field B_μ , we define the field strength tensor $B_{\mu\nu}$.

Where

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (2.11)$$

Hence the gauge field B_μ is included in the lagrangian with addition of the term

$$L_B = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} \quad (2.12)$$

we want introduce this term but preserve of the action under variation $\delta\delta = 0$. Once more the gauge field B_μ forces as to introduce extra term into the derivative in order to maintain covariance. This is possible if we take

$$\partial_\mu \rightarrow \partial_\mu + i \frac{g_B}{2} B_\mu$$

where g_B is a coupling constant associated with gauge field B_μ , in a similar fashion, we will define a field strength tensor associated with gauge field W_μ^1 , W_μ^2 and W_μ^3 , first let's consider SU(2) transformations. We now reintroduce the Pauli matrices an anticipation of including SU(2) as

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Since these are just the Pauli matrices, We have SU(2) algebra.

$$[\tau_i, \tau_j] = 2i \varepsilon_{ijk} \tau_k$$

The τ_i generators define weak isospin space. We now use them to consider a SU(2) transformation of the form

$$U(\alpha) = \exp\left(\frac{i\alpha_j \tau_j}{2}\right)$$

The right-handed electron spinor e_R is invariant under SU(2) transformation

$$\Psi_R \rightarrow \hat{\Psi}_R = U(\alpha) \Psi_R = \Psi_R$$

However, the left-handed spinor transforms in the usual way as

$$\Psi_L \rightarrow \hat{\Psi}_L = e^{\frac{-\tau\alpha}{2}} \Psi_L$$

The reason for these transformation properties is that right-handed electron e_R does not carry any weak isospin charge ($\vec{I}_R = I_R^3 = 0$), Which is associated with SU(2) transformation, hence it does not couple to the W_μ^1 , W_μ^2 and W_μ^3 fields.

The electron neutrino and left-handed electron state do carry isospin so we need to apply SU(2) in that case. In matrix form, the SU(2) can be written as

$$\begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e \\ \dot{e}_L \\ \dot{e}_R \end{pmatrix} = \begin{pmatrix} e^{\frac{-i\tau\alpha}{2}} & 0 & 0 \\ 0 & e^{\frac{-i(\tau.\alpha)}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix}$$

Now let's consider the field strength tensor corresponding to the gauge fields W_μ^1 , W_μ^2 and W_μ^3 it assumes the form

$$F_{\mu\nu}^l = \partial_\mu W_\nu^l - \partial_\nu W_\mu^l - g_w \varepsilon^{lmn} W_\mu^m W_\nu^n \quad (2.13)$$

We add the field tensor to the Lagrangian by summing up $F_{\mu\nu}^l F^{\mu\nu}$ over $l = 1, 2, 3$. To include each of the gauge fields W_μ^1 , W_μ^2 and W_μ^3 . That is we take the trace and can write the contribution to the lagrangian as

$$L_W = -\frac{1}{8} \text{Tr} (F_{\mu\nu} F^{\mu\nu}) \quad (2.14)$$

To keep derivative covariant, now we need to add an extra term to account for the presence of Eq (2.13) in the Lagrangian. This is done by adding the following term to the derivative

$$i g_\mu \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu = i \frac{g_\mu}{2} (\tau_1 W_\mu^1 + \tau_2 W_\mu^2 + \tau_3 W_\mu^3)$$

Since the right-handed lepton field does not participate in the interaction involving weak isospin, this term is not added to the derivative in that case. We only add the term to account for the presence of the gauge field B_μ [7].

Hence

$$i \bar{\Psi}_R \partial^\mu \partial_\mu \Psi_R \rightarrow i \bar{\Psi}_R \partial^\mu (\partial_\mu + i \frac{g_B}{2} B_\mu) \Psi_R$$

For the left-handed field, we have

$$i \bar{\Psi}_L \partial^\mu \partial_\mu \Psi_L \rightarrow i \bar{\Psi}_L \partial^\mu (\partial_\mu + i \frac{g_B}{2} B_\mu + i g_\mu \frac{\vec{\tau}}{2} \vec{W}_\mu) \Psi_L$$

So the total leptonic portion of the Lagrangian is

$$\begin{aligned}
L_{lepton} = & i \bar{\Psi}_R \partial^\mu (\partial_\mu + i \frac{g_B}{2} B_\mu) \Psi_R \\
& + i \bar{\Psi}_L \partial^\mu (\partial_\mu + i \frac{g_B}{2} B_\mu + i \frac{g_w}{2} \tau_w) \Psi_L
\end{aligned} \tag{2.15}$$

The total Lagrangian includes the gauge field Lagrangian as

$$\begin{aligned}
L = & L_{fermions} + L_{gauge} \\
= & i \bar{\Psi}_R \partial^\mu (\partial_\mu + i \frac{g_B}{2} B_\mu) \Psi_R + i \bar{\Psi}_L \partial^\mu (\partial_\mu + i \frac{g_B}{2} B_\mu + i \frac{g_w}{2} \tau_w) \Psi_L \\
& - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{8} T_r(F_{\mu\nu} F^{\mu\nu})
\end{aligned} \tag{2.16}$$

(2-5-5) Weak mixing or Weinberg angle:

In the following sections we will see that it is convenient to relate the coupling constant g_B and g_w in the following way

$$\tan \theta_w = \frac{g_w}{g_b} \tag{2.17}$$

The angle θ_w is often called the Weinberg angle. It is also sometimes called the weak mixing angle. The reason is that it mixes the gauge fields to give

$$A_\mu = B_\mu \cos \theta_w + W_\mu^3 \sin \theta_w \tag{2.18}$$

$$Z_\mu = -B_\mu \sin \theta_w + W_\mu^3 \cos \theta_w \tag{2.19}$$

We can view this as a rotation, writing the relationship in matrix form:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = R(\theta_w) \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} \cos \theta_w & \sin \theta_w \\ -\sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

The gauge field A_μ is nothing other than the electromagnetic vector potential which couples to the proton to the theory, we will explore this in more details after we introduce the Higgs mechanism.

Chapter Three

Higgs mechanism

(3-1) Introduction:

In this chapter, we will study and discuss quantitatively and qualitatively the Higgs boson, in particular we emphasize the Higgs mechanism idea sometime refer as the spontaneous symmetry breaking. Furthermore, we also present and show how Higgs particle interact with other particle to give them masses.

(3-2) spontaneous symmetry breaking:

At this point we have put together a theory describing two leptons, the electron and its corresponding neutrino, and four gauge bosons. All the particle described so far are mass less. It's time to introduce mass into the theory and this will be done using the symmetry breaking method. We will introduce a Higgs field that will force the gauge bosons to acquire mass we also we will introduce the photon field explicitly, so we can present a unified picture of the electroweak interaction [8].

$$\Phi = \begin{pmatrix} \Phi^A \\ \Phi^B \end{pmatrix} \quad (3.1)$$

Each component is a complex scalar field and is written as

$$\Phi^A = \frac{\Phi_3 + i\Phi_4}{\sqrt{2}} \quad , \quad \Phi^B = \frac{\Phi_1 + i\Phi_2}{\sqrt{2}}$$

So the Higgs field is composed of four real scalar fields. we see that

$$\begin{aligned} \Phi^\dagger \Phi &= (\Phi^A)^\dagger (\Phi^A) + (\Phi^B)^\dagger (\Phi^B) \\ &= \frac{1}{2} (\Phi_1^2 + \Phi_2^2 + \Phi_3^2 + \Phi_4^2) \end{aligned}$$

The Higgs carriers charges of the weak interaction specifically

$$Y_\Phi = +1 \quad , \quad I_\Phi = \frac{1}{2}$$

The SU(2) transform is given by

$$U(\alpha) = e^{i\alpha_j \tau_j / 2}$$

We have a gauge freedom that can be exploited to simplify the form of the Higgs field. This can be done by insisting that each of the " angles " are functions of space time

$$\alpha_j = \alpha_j(x)$$

Because we want a local symmetry

$$\Phi^A = 0 \quad ; \quad \Phi^B = \Phi_0 + \frac{h(\alpha)}{\sqrt{2}}$$

$$\Phi = \begin{pmatrix} 0 \\ \Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \end{pmatrix} \quad (3.2)$$

The parameter Φ_0 and field $h(\alpha)$ are both real, Φ_0 allows us to break the symmetry. Instead of taking the ground state to be ($\Phi \rightarrow 0$) we take to be

$$\Phi_G = \begin{pmatrix} 0 \\ \Phi_0 \end{pmatrix} \quad (3.3)$$

(3-3) Giving mass to the lepton fields:

Dirac Lagrangian consisting of left-handed and right-handed fields with mass(m) is written as

$$L = i \bar{\Psi}_L \partial^\mu \partial_\mu \Psi_L + i \bar{\Psi}_R \partial^\mu \partial_\mu \Psi_R - m(\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L) \quad (3.4)$$

Hence a mass term in the Lagrangian is of the form

$$- m(\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L)$$

Where m is a scalar. In Weinberg-Salam theory an interaction term (known as Yukawa term) is introduced that couples the matter fields to the Higgs field.

The Yukawa coupling (Y) which defines the strength of the interaction between the Higgs field and the electron-lepton fields. The Lagrangian is

$$L_{Yukawa} = -Y (\bar{\Psi}_L \Phi \Psi_R + \bar{\Psi}_R \Phi^\dagger \Psi_L) \quad (3.5)$$

Let's look at each term

$$\bar{\Psi}_L \Phi = (\bar{\nu}_e \quad \bar{e}_L) \begin{pmatrix} \Phi^A \\ \Phi^B \end{pmatrix} = \bar{\nu}_e \Phi^A + \bar{e}_L \Phi^B$$

Now, using the gauge choice hed to the form of the higgs field, the neutrino term drops out

$$\bar{\Psi}_L \Phi = (\bar{\nu}_e \quad \bar{e}_L) \begin{pmatrix} \Phi^A \\ \Phi^B \end{pmatrix} = \bar{\nu}_e \Phi^A + \bar{e}_L \Phi^B = \bar{e}_L (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}})$$

using

$$\Psi_R = \begin{pmatrix} 0 \\ e_R \end{pmatrix}$$

we have

$$\bar{\Psi}_L \Phi \Psi_R = \bar{e}_L (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) e_R$$

$$\bar{\Psi}_R \Phi^\dagger \Psi_L = (0 \quad \bar{e}_R) \begin{pmatrix} 0 & \Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$$

$$= (0 \quad \bar{e}_R) (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) e_L$$

$$= \bar{e}_R (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) e_L$$

$$L_{Yukawa} = -Y (\bar{\Psi}_L \Phi \Psi_R + \bar{\Psi}_R \Phi^\dagger \Psi_L)$$

$$= -Y (\bar{e}_L (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) e_R + \bar{e}_R (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) e_L)$$

$$= -Y \Phi_0 (\bar{e}_L e_R + \bar{e}_R e_L) - Y \frac{h(\alpha)}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) \quad (3.6)$$

Looking for the mass we ignore the second term because a mass term in the Lagrangian is multiplied by an overall scalar (a number).

$$L_{mass} = -Y \Phi_0 (e_L e_R + \bar{e}_R e_L) \quad (3.7)$$

$$\therefore m = Y \Phi_0$$

(3-4) Gauge masses:

Now we will see how the Higgs mechanism gives mass to the gauge bosons. The gauge bosons will acquire a mass through the action of the covariant derivative on the Higgs field. The field W_μ^1 and W_μ^2 are electrically charged. And can be combined into the physical fields as shown here

$$W_\mu^+ = \frac{W_\mu^1 - i W_\mu^2}{\sqrt{2}}$$

$$W_\mu^- = \frac{W_\mu^1 + i W_\mu^2}{\sqrt{2}}$$

We have covariant derivative given by

$$D_\mu = \partial_\mu + \frac{i g_B}{2} + \frac{i g_w}{2} \vec{\tau} \cdot \vec{W}_\mu$$

Now we apply the covariant derivative to the Higgs field

$$D_\mu \Phi = \partial_\mu \Phi + \frac{i g_B}{2} + \frac{i g_w}{2} \vec{\tau} \cdot \vec{W}_\mu \Phi \quad (3.8)$$

Notice that

$$\begin{aligned} \vec{\tau} \cdot \vec{W}_\mu &= \tau_1 W_\mu^1 + \tau_2 W_\mu^2 + \tau_3 W_\mu^3 \\ &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} W_\mu^1 + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} W_\mu^2 + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} W_\mu^3 \\ &= \begin{pmatrix} W_\mu^3 & W_\mu^1 - i W_\mu^2 \\ W_\mu^1 + i W_\mu^2 & -W_\mu^3 \end{pmatrix} \end{aligned}$$

So

$$\begin{aligned} \frac{i g_w}{2} \vec{\tau} \cdot \vec{W}_\mu \Phi &= \frac{i g_w}{2} \begin{pmatrix} w_\mu^3 & w_\mu^1 - i w_\mu^2 \\ w_\mu^1 + i w_\mu^2 & -w_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ \Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \end{pmatrix} \\ &= \frac{i g_w}{2} \begin{pmatrix} \sqrt{2} w_\mu^+ (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) \\ -w_\mu^3 (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) \end{pmatrix} \end{aligned}$$

The first term in Eq (3.8) just gives the ordinary derivative of the Higgs field.

$$\partial_\mu \begin{pmatrix} 0 \\ (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \partial_\mu h(\alpha) \end{pmatrix}$$

This term is the kinetic energy term and does not contribute to the generation of the boson masses, so we won't worry about it. but putting together we have

$$D_\mu \Phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \partial_\mu h(\alpha) \end{pmatrix} + \frac{i g_w}{2} \begin{pmatrix} \sqrt{2} w_\mu^+ (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) \\ -w_\mu^3 (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) \end{pmatrix} + \frac{i g_B}{2} B_\mu \begin{pmatrix} 0 \\ (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) \end{pmatrix}$$

To find the mass terms, we calculate

$$(D_\mu \Phi)^\dagger D^\mu \Phi$$

And only keep terms of quadratic order.

Now

$$\begin{aligned} (D_\mu \Phi)^\dagger &= \left(0 \quad \frac{h(\alpha)}{\sqrt{2}} \right) - \frac{i g_w}{2} \left(\sqrt{2} w_\mu^- (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) \quad -w_\mu^3 (\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}) \right) \\ &\quad - \frac{i g_B}{2} B_\mu \left(0 \quad \Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \end{aligned}$$

The first term in the product $(D_\mu \Phi)^\dagger D^\mu \Phi$ is

$$\frac{1}{2} (\partial_\mu h)^2$$

The next term we get is

$$\begin{aligned}
& - \frac{i g_w}{2} \left(\sqrt{2} w_\mu^- \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \quad - w_\mu^3 \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \right) \\
& \quad - \frac{i g_w}{2} \begin{pmatrix} \sqrt{2} w_\mu^+ \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \\ - w_\mu^3 \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \end{pmatrix} \\
& = 2 \frac{g_w^2}{4} w_\mu^- w_\mu^+ \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right)^2 + \frac{g_w^2}{4} w_\mu^3 w_\mu^3 \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right)
\end{aligned}$$

And then we have a cross term

$$\begin{aligned}
& - \frac{i g_w}{2} \left(\sqrt{2} w_\mu^- \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \quad - w_\mu^3 \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \right) i \frac{g_B}{2} B^\mu \begin{pmatrix} 0 \\ \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \end{pmatrix} \\
& = - \frac{g_w g_B}{2} w_\mu^3 B^\mu \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right)^2
\end{aligned}$$

The second cross term is

$$\begin{aligned}
& \frac{-i g_B}{2} B_\mu \begin{pmatrix} 0 \\ \Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \end{pmatrix} \frac{i g_w}{2} \begin{pmatrix} \sqrt{2} w_\mu^+ \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \\ - w_\mu^3 \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \end{pmatrix} \\
& = - \frac{g_B g_w}{4} B_\mu w_\mu^3 \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right)^2
\end{aligned}$$

The final quadratic term of interest is

$$\begin{aligned}
& \frac{-i g_B}{2} B_\mu \begin{pmatrix} 0 \\ \Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \end{pmatrix} \frac{i g_B}{2} \begin{pmatrix} 0 \\ \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right) \end{pmatrix} \\
& = \frac{g_B^2}{4} B_\mu B^\mu \left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}} \right)^2
\end{aligned}$$

We wish to write these expression in terms of the physical fields using the Weinberg angle.

$$\begin{pmatrix} B_\mu \\ w_\mu^3 \end{pmatrix} = \mathbf{R}(-\theta_w) \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

$$B_\mu = A_\mu \cos\theta_w - Z_\mu \sin\theta_w$$

$$W_\mu^3 = A_\mu \sin\theta_w + Z_\mu \cos\theta_w$$

Keeping only the leading term

$$\left(\Phi_0 + \frac{h(\alpha)}{\sqrt{2}}\right)^2$$

Which is only term that gives scalar (number), we find

$$\begin{aligned} & \frac{g_w^2}{2} W_\mu^- W^{+\mu} \Phi_0^2 + \frac{g_w^2}{4} W_\mu^3 W^{3\mu} \Phi_0^2 \\ &= \frac{g_w^2}{2} W_\mu^- W^{+\mu} \Phi_0^2 + \frac{g_w^2}{4} \Phi_0^2 (A_\mu \sin\theta_w + Z_\mu \cos\theta_w) (A^\mu \sin\theta_w + Z^\mu \cos\theta_w) \\ &= \frac{g_w^2}{2} W_\mu^- W^{+\mu} \Phi_0^2 + \frac{g_w^2}{4} \Phi_0^2 (A_\mu A^\mu \sin^2\theta_w \\ &+ A_\mu Z^\mu \sin\theta_w \cos\theta_w + Z_\mu A^\mu \sin\theta_w \cos\theta_w + Z_\mu Z^\mu \cos^2\theta_w) \end{aligned} \quad (3.9)$$

The next term becomes

$$\begin{aligned} & \frac{-g_B g_w}{4} W_\mu^3 B^\mu \Phi_0^2 \\ &= \frac{-g_w g_B}{4} (A_\mu \sin\theta_w + Z_\mu \cos\theta_w) (A^\mu \cos\theta_w - Z^\mu \sin\theta_w) \Phi_0^2 \\ &= \frac{-g_w g_B}{4} [A_\mu A^\mu \cos\theta_w \sin\theta_w - A_\mu Z^\mu \sin^2\theta_w + Z_\mu A^\mu \cos^2\theta_w \\ &\quad - Z_\mu Z^\mu \cos\theta_w \sin\theta_w] \end{aligned} \quad (3.10)$$

The second cross term becomes

$$\begin{aligned} & \frac{-g_B g_w}{4} B^\mu W^{3\mu} \Phi_0^2 \\ &= \frac{-g_B g_w}{4} (A^\mu \cos\theta_w - Z^\mu \sin\theta_w) (A^\mu \sin\theta_w + Z^\mu \cos\theta_w) \Phi_0^2 \\ &= \frac{-g_w g_B}{4} [A_\mu A^\mu \cos\theta_w \sin\theta_w + A_\mu Z^\mu \cos^2\theta_w - Z_\mu A^\mu \sin^2\theta_w \\ &\quad - Z_\mu Z^\mu \sin\theta_w \cos\theta_w] \Phi_0^2 \end{aligned} \quad (3.11)$$

And the final term is

$$\begin{aligned}
& \frac{g_B^2}{4} B_\mu B^\mu \Phi_0^2 \\
&= \frac{g_B^2}{4} (A^\mu \cos\theta_w - Z^\mu \sin\theta_w) (A^\mu \sin\theta_w + Z^\mu \cos\theta_w) \Phi_0^2 \\
&= \frac{g_B^2}{4} [A_\mu A^\mu \cos^2\theta_w - A_\mu Z^\mu \cos\theta_w \sin\theta_w - Z_\mu A^\mu \sin\theta_w \cos\theta_w \\
&\quad + Z_\mu Z^\mu \sin^2\theta_w] \Phi_0^2 \tag{3.12}
\end{aligned}$$

Now we add up Eq (3.9) through Eq (3.12). We are looking for mass terms in the field, so we ignore all mixed term that describe interaction like $A_\mu Z^\mu$. Let's look at each combination of the field.

We have

$$\begin{aligned}
& A_\mu A^\mu \left[\frac{g_B^2}{4} \cos^2\theta_w - \frac{2g_B g_w}{4} \sin\theta_w \cos\theta_w + \frac{g_w^2}{4} \sin^2\theta_w \right] \\
&= A_\mu A^\mu \left[\frac{g_B^2}{4} \cos^2\theta_w + \sin^2\theta_w - \frac{g_B g_w}{2} \sin\theta_w \cos\theta_w \right]
\end{aligned}$$

The mixing angle describes how the forces mix as you can see from the figure below, if ($\theta_w = 0$), then we have pure coupling to the W boson, and no coupling to Z boson, at non zero θ_w less than 90° indicates coupling to both fields (thus the term weak mixing angle) as shown in the following illustration [7,8].

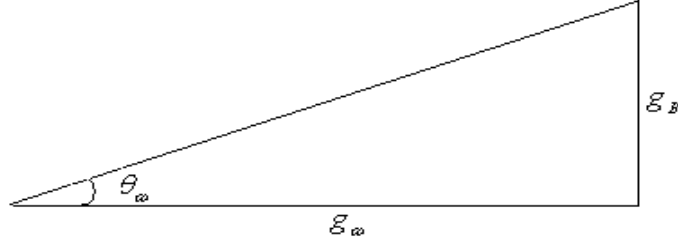


Figure (3-1): weak mixing angle.

From figure (3.1) we see that:

$$\cos\theta_w = \frac{g_w}{\sqrt{g_B^2 + g_w^2}}$$

$$\sin\theta_w = \frac{g_B}{\sqrt{g_B^2 + g_w^2}}$$

And this because

$$\begin{aligned} & A_\mu A^\mu \left[\frac{g_B^2}{4} \cos^2\theta_w + \frac{g_w^2}{4} \sin^2\theta_w - \frac{g_B g_w}{4} \sin\theta_w \cos\theta_w \right] \\ &= A_\mu A^\mu \left[\frac{g_B^2}{4} \frac{g_w^2}{(g_B^2 + g_w^2)} + \frac{g_B^2}{4} \frac{g_B^2}{(g_B^2 + g_w^2)} - \frac{g_B g_w}{2} \frac{g_B}{\sqrt{g_B^2 + g_w^2}} \frac{g_w}{\sqrt{g_B^2 + g_w^2}} \right] \\ &= 0 \end{aligned}$$

This tells us that A_μ is a massless field. In fact, it couples to the electric charge. So we know this is our photon field.

Now for the Z field we get

$$\begin{aligned}
& Z_\mu Z^\mu \left[\frac{g_w^2}{4} \cos^2 \theta_w + \frac{g_B^2}{4} \sin^2 \theta_w + \frac{g_w g_B}{2} \sin \theta_w \cos \theta_w \right] \Phi_0^2 \\
&= Z_\mu Z^\mu \left[\frac{g_w^2}{4} \frac{g_w^2}{g_w^2 + g_B^2} + \frac{g_B^2}{4} \frac{g_B^2}{g_B^2 + g_w^2} + \frac{g_w g_B}{2} \frac{g_B}{\sqrt{g_B^2 + g_w^2}} \frac{g_w}{\sqrt{g_B^2 + g_w^2}} \right] \Phi_0^2 \\
&= Z_\mu Z^\mu \left[\frac{g_B^2 g_B^2 + g_w^2 g_w^2 + 2 g_w^2 g_B^2}{4(g_w^2 + g_B^2)} \right] \Phi_0^2 \\
&= Z_\mu Z^\mu \left[\frac{(g_B^2 + g_w^2)^2}{4(g_w^2 + g_B^2)} \right] \Phi_0^2 \\
&= Z_\mu Z^\mu \left[\frac{1}{4} (g_B^2 + g_w^2) \right] \Phi_0^2
\end{aligned}$$

Therefore the mass of the Z particle is

$$M_Z = \left(\frac{\sqrt{g_B^2 + g_w^2}}{2} \Phi_0 \right) \quad (3.13)$$

Chapter Four

Calculations and Results

(4-1) Introduction:

This chapter will present our calculations and results in details for the Higgs decay rates into gauge bosons and fermions ($H \rightarrow WW$) and ($H \rightarrow ff$) by making use of so called the standard model theory, then we summarised our numerical calculations in the tables below.

We will utilise the following equations for our calculations:

$$\Gamma(H \rightarrow WW) = \frac{m_h^3 G_F}{8\sqrt{2} \pi} \left(1 - \frac{m_W^2}{m_h^2}\right)^{\frac{1}{2}} \left(1 - 4 \frac{m_W^2}{m_h^2} + 12 \frac{m_W^4}{m_h^4}\right), \quad (4-1)$$

$$\Gamma(H \rightarrow ZZ) = \frac{m_h^3 G_F}{16\sqrt{2} \pi} \left(1 - \frac{m_Z^2}{m_h^2}\right)^{\frac{1}{2}} \left(1 - 4 \frac{m_Z^2}{m_h^2} + 12 \frac{m_Z^4}{m_h^4}\right) \quad (4-2)$$

And

$$\Gamma(H \rightarrow ff) = \frac{3m_f^2 m_h G_F}{4\sqrt{2} \pi} \left(1 - \frac{m_f^2}{m_h^2}\right)^{\frac{3}{2}}. \quad (4-3)$$

As can be seen from the above equations, for the higgs to decay to gauge bosons (W and Z) its mass should be bigger or equal twice the mass of these particles at least as well as other fermions. As such we take as input parameters $M_W = 80.4\text{GeV}$, $M_Z = 91.2\text{GeV}$ and $G_F = 1.16637 \times 10^{-5}\text{GeV}^{-2}$ with $m_h = (100 - 400)\text{GeV}$.

Note that the Higgs decay to bb as highlighted in the table (4-4) is very small and can be safely neglected in our calculation, only the top quark is the dominant contribution because its mass is bigger than other quarks and leptons. And we have double checked for the Higgs decay to tau lepton and found that the decay rate is also too small compare to top quark [10].

(4-2) Higgs decay to gauge bosons:

$$L_{H \rightarrow VV} = g M_V h V^\mu V_\mu \quad (4.4)$$

where

V is vector representing gauge boson

$$(V = W^\pm, Z^0)$$

For (W^\pm) :

$$L_{H \rightarrow WW} = g M_W h W^\mu W_\mu \quad (4.5)$$

$H \rightarrow w^+ w^-$:

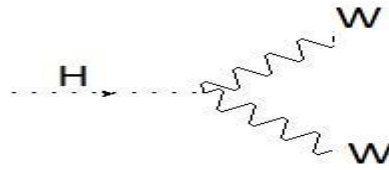


Figure (4.1): Higgs decay to WW gauge boson

$$M = (-i g) M_W (\varepsilon_1 * \varepsilon_2) \quad (4.6)$$

Then

$$\begin{aligned} \sum_{spin} |M|^2 &= g^2 M_W^2 \sum_{spin} (\varepsilon_1 * \varepsilon_2)^2 \\ &= g^2 M_W^2 \left[-g^{\mu\nu} + \frac{K^\mu K^\nu}{M_W^2} \right] \left[-g_{\mu\nu} + \frac{K'_\mu K'_\nu}{M_W^2} \right] \\ &= g^2 M_W^2 \left[2 + \frac{(K * k')^2}{M_W^2} \right] \end{aligned} \quad (4.7)$$

But

$$M_w^2 = (k * k')^2 \quad (4.8)$$

$$k * k' = \frac{1}{2} (M_H^2 - 2M_w^2)$$

Then

$$\sum_{spin} |M|^2 = \frac{2G_f}{\sqrt{2}} M_H^4 \left[1 - \frac{4M_w^2}{M_H^2} + \frac{12M_w^4}{M_H^4} \right] \quad (4.9)$$

The decay rate is given by

$$\Gamma = \frac{1}{2M_H} \int (2\pi)^4 \delta^4 (p - k - k') \frac{d^3k'}{(2\pi)^3} \frac{d^3K}{(2\pi)^3} \frac{1}{2E} \frac{1}{2E'} (\sum_{spin} |M|^2) \quad (4.10)$$

In the rest frame the phase space is

$$\rho = \int (2\pi)^4 \delta^4 (p - k - k') \frac{d^3k'}{(2\pi)^3} \frac{1}{2E'} \frac{d^3K}{(2\pi)^3} \frac{1}{2E} \quad (4.11)$$

$$= \frac{1}{(2\pi)^3} \int \delta (M_H - E - E') \frac{d^3k}{2E2E'} \quad (4.12)$$

$$d^3k = 4\pi k^2 dk = 4\pi kE dE \quad (4.13)$$

We get

$$\rho = \frac{1}{4\pi} \int \delta (M_H - 2E) \frac{k dE}{2E} = \frac{k}{4\pi M_H} \quad (4.14)$$

The momentum (k) can be calculated as follow:

$$E = \frac{1}{2} M_H = (k^2 + M_w^2)^{\frac{1}{2}} \quad (4.15)$$

$$k = \frac{1}{2} (M_H^2 - 4M_w^2)^{\frac{1}{2}} \quad (4.16)$$

The phase space is then give by

$$\rho = \frac{1}{8\pi} \left(1 - \frac{4M_w^2}{M_H^2}\right)^{\frac{1}{2}} \quad (4.17)$$

Therefore

$$\therefore \Gamma_{(H \rightarrow w^+ w^-)} = \frac{M_H^3 G_f}{8\sqrt{2}\pi} \left(1 - \frac{4M_w^2}{M_H^2}\right)^{\frac{1}{2}} \left(1 - \frac{4M_w^2}{M_H^2} + \frac{12M_w^4}{M_H^4}\right) \quad (4.18)$$

For (Z^0) :

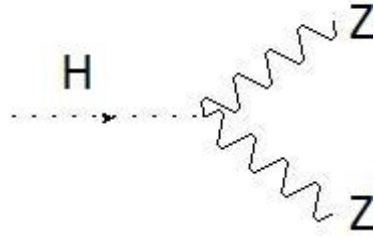


Figure (4-2): Higgs decay to ZZ gauge boson

$H \rightarrow ZZ$:

$$M = (-i g) \frac{M_Z}{\cos \theta_w} (\varepsilon_1 * \varepsilon_2) \quad (4.19)$$

Then

$$\sum_{spin} |M|^2 = \frac{g^2 M_Z^2}{\cos^2 \theta_w} \sum_{spin} (\varepsilon_1 * \varepsilon_2)^2 \quad (4.20)$$

$$= \frac{g^2 M_Z^2}{\cos^2 \theta_w} \left[2 + \frac{k \cdot k'}{M_Z^2} \right]$$

$$= \frac{g^2 M_Z^2}{4 \cos^2 \theta_w} \left[M_H^2 - 4 M_H^2 M_Z^2 + 12 M_Z^4 \right]$$

$$= \frac{g^2 M_Z^2}{4 \cos^2 \theta_w} M_H^4 \left[1 - \frac{4 M_Z^2}{M_H^2} + \frac{12 M_Z^4}{M_H^4} \right] \quad (4.21)$$

Now the phase space having an extra factor of (12) because of the identical particles in decay product.

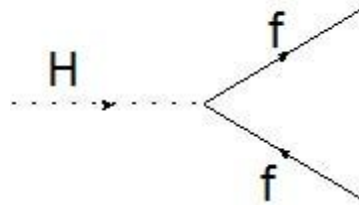
Therefore

$$\therefore \Gamma_{(H \rightarrow ZZ)} = \frac{M_H^3 G_f}{16 \sqrt{2} \pi} \left(1 - \frac{4 M_Z^2}{M_H^2} \right)^{\frac{1}{2}} \left(1 - \frac{4 M_Z^2}{M_H^2} + \frac{12 M_Z^4}{M_H^4} \right) \quad (4.25)$$

(4-3) Higgs decay to Fermions:

$$L_{H \rightarrow f\bar{f}} = \frac{g}{2} \frac{M_f}{M_w} h \bar{f} f \quad (4.26)$$

$H \rightarrow f\bar{f}$:



Figure(4.3): Higgs decay to fermions

$$M = -i \frac{M_f}{V} \bar{V}(k') V(k) \quad (4.27)$$

Then

$$\sum_{spin} |M|^2 = \frac{M_f^2}{V^2} \text{Tr} [(k + M_f) (k' - M_f)] \quad (4.28)$$

Use the trace technology

$$\text{Tr} (\gamma_\mu \gamma_\nu) = 4 g_{\mu\nu} \quad (2.29)$$

$$\text{Tr} (\gamma_\mu) = 0 \quad (4.30)$$

So

$$\sum_{spin} |M|^2 = 4 \frac{M_f^2}{V^2} (k * k' - M_f^2) \quad (4.31)$$

We have

$$M_H^2 = (k + k')^2 \quad (4.32)$$

$$k * k' = \frac{1}{2} (M_H^2 - 2M_f^2) \quad (4.33)$$

$$\begin{aligned}
\Sigma_{spin} |M|^2 &= 4 \frac{M_f^2}{V^2} \left(\frac{1}{2} M_H^2 - 2M_f^2 \right) \\
&= \frac{4}{2} \frac{M_f^2}{V^2} (M_H^2 - 4M_f^2) \\
&= 2 \frac{M_f^2}{V^2} M_H^2 \left(1 - \frac{M_f^2}{M_H^2} \right)
\end{aligned} \tag{4.34}$$

The phase space is given by

$$\rho = \frac{1}{8\pi} \left(1 - \frac{4M_f^2}{M_H^2} \right)^{\frac{1}{2}} \tag{4.35}$$

Therefore

$$\therefore \Gamma_{(H \rightarrow f\bar{f})} = \frac{M_H M_f^2 G_f}{4\sqrt{2} \pi} \left(1 - \frac{4M_f^2}{M_H^2} \right)^{\frac{3}{2}} (N_c) \tag{4.36}$$

Where

$$N_c = \begin{cases} 3 & \text{for quarks} \\ 1 & \text{for leptons} \end{cases}$$

(4-1): Decay rate and Branching ratio for $(H \rightarrow w^+w^-)$

M_{Higgs} (GeV)	$\Gamma_{(H \rightarrow w^+w^-)}$ (GeV)	Branching ratio
100	–	–
150	–	–
200	1.1	0.7
250	62.84	0.71
300	5.9	0.7
350	10.2	0.7
400	16.9	0.6

Table(4-2): Decay rate and Branching ratio for ($H \rightarrow zz$)

M_{Higgs} (Gev)	$\Gamma_{(H \rightarrow zz)}$ (GeV)	Branching ratio
100	–	–
150	–	–
200	0.4	0.26
250	1.2	0.3
300	2.6	0.3
350	4.5	0.3
400	6.9	0.26

Table(4-3): Decay rate Branching ratio for ($H \rightarrow tt$)

M_{Higgs} (GeV)	$\Gamma_{(H \rightarrow tt)}$ (GeV)	Branching ratio
100	-	-
150	-	-
200	-	-
250	-	-
300	-	-
350	-	-
400	2.15	0.08

Table(4-4): Decay rate and Branching ratio for ($H \rightarrow bb$)

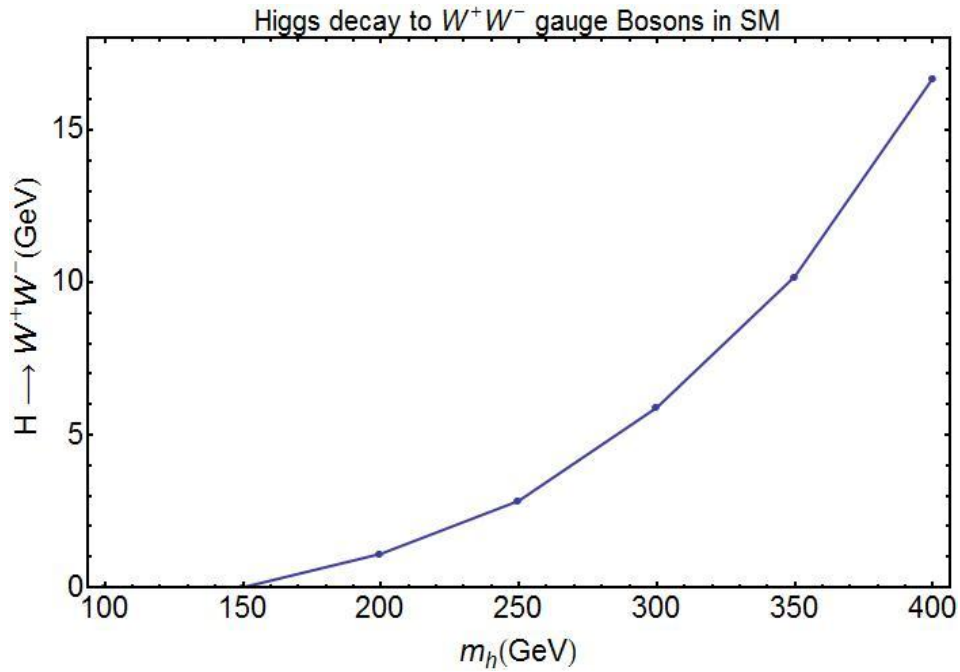
M_{Higgs} (Gev)	$\Gamma_{(H \rightarrow bb)}$ (GeV)	Branching ratio
100	$4 * 10^{-3}$	$9.5 * 10^{-7}$
150	$6 * 10^{-3}$	$9.5 * 10^{-7}$
200	$8 * 10^{-3}$	$5.3 * 10^{-3}$
250	0.01	$2.5 * 10^{-3}$
300	0.0119	$1.4 * 10^{-3}$
350	0.014	$9.5 * 10^{-3}$
400	0.0158	$6.2 * 10^{-3}$

Table(4-5): Decay rate branching ratio for ($H \rightarrow \mu\mu$)

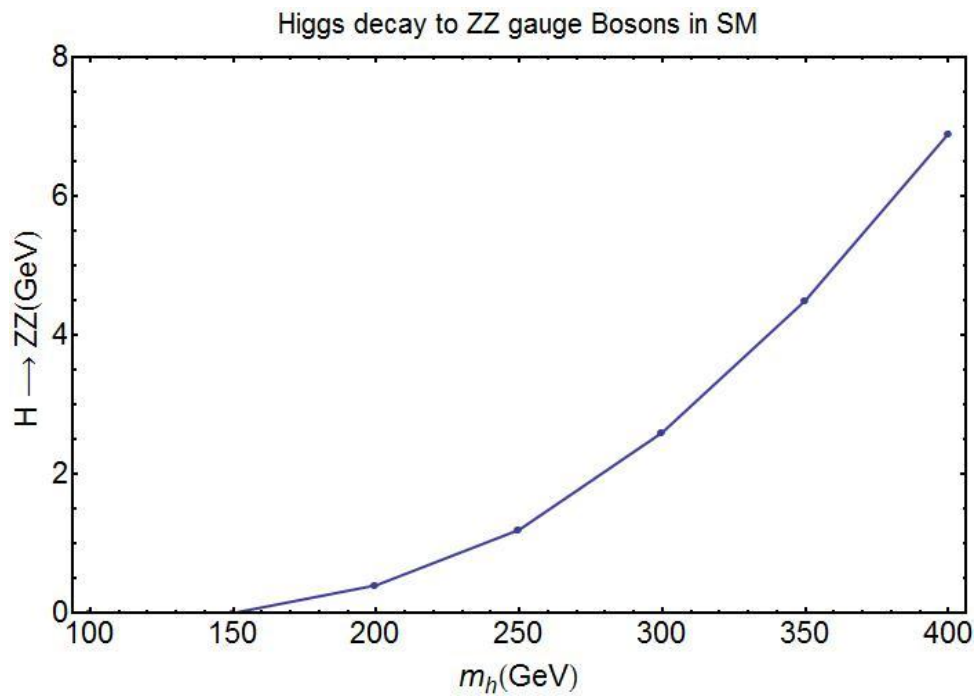
M_{Higgs} (GeV)	$\Gamma_{(H \rightarrow \mu\mu)}$ (GeV)	Branching ratio
100	$7.4 * 10^{-4}$	$1.8 * 10^{-10}$
150	$1.1 * 10^{-6}$	$1.75 * 10^{-7}$
200	$1.5 * 10^{-6}$	$001 * 10^{-6}$
250	$1.8 * 10^{-6}$	$4.5 * 10^{-7}$
300	$2.2 * 10^{-6}$	$2.6 * 10^{-7}$
350	$2.6 * 10^{-6}$	$1.8 * 10^{-7}$
400	$3 * 10^{-6}$	$1.2 * 10^{-7}$

Table(4-6): Decay rate and Branching ratio for ($H \rightarrow \tau\tau$)

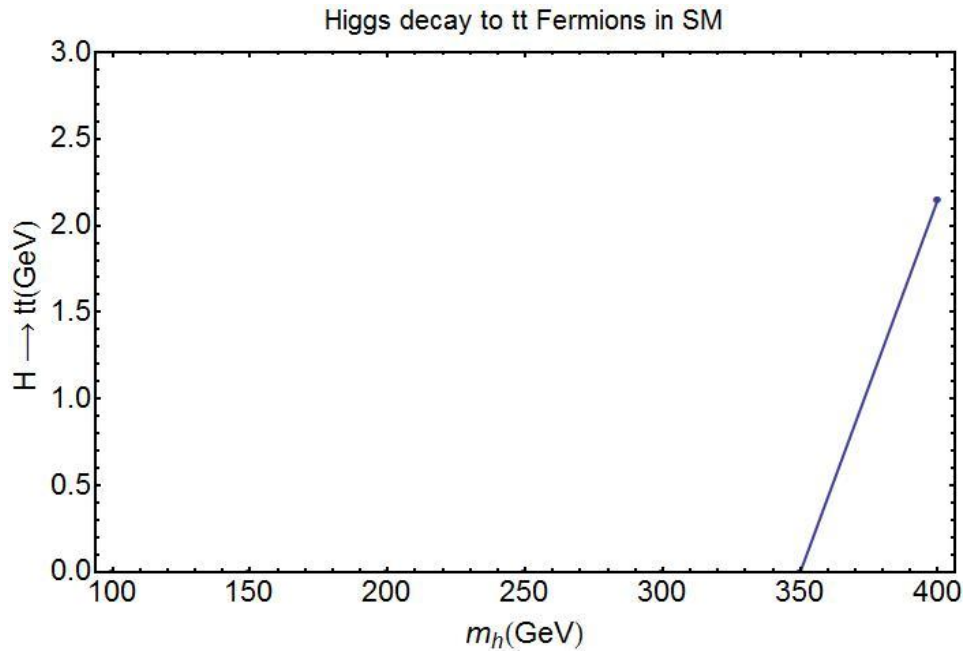
M_{Higgs}	$\Gamma_{(H \rightarrow \tau\tau)}$ (GeV)	Branching ratio
100	$2.2 * 10^{-4}$	$5 * 10^{-8}$
150	$3.1 * 10^{-4}$	$4.9 * 10^{-8}$
200	$4.2 * 10^{-4}$	$2.8 * 10^{-4}$
250	$5.2 * 10^{-4}$	$1.3 * 10^{-4}$
300	$6.2 * 10^{-4}$	$7.3 * 10^{-5}$
350	$7.3 * 10^{-4}$	$4.9 * 10^{-5}$
400	$8.3 * 10^{-4}$	$3.2 * 10^{-5}$



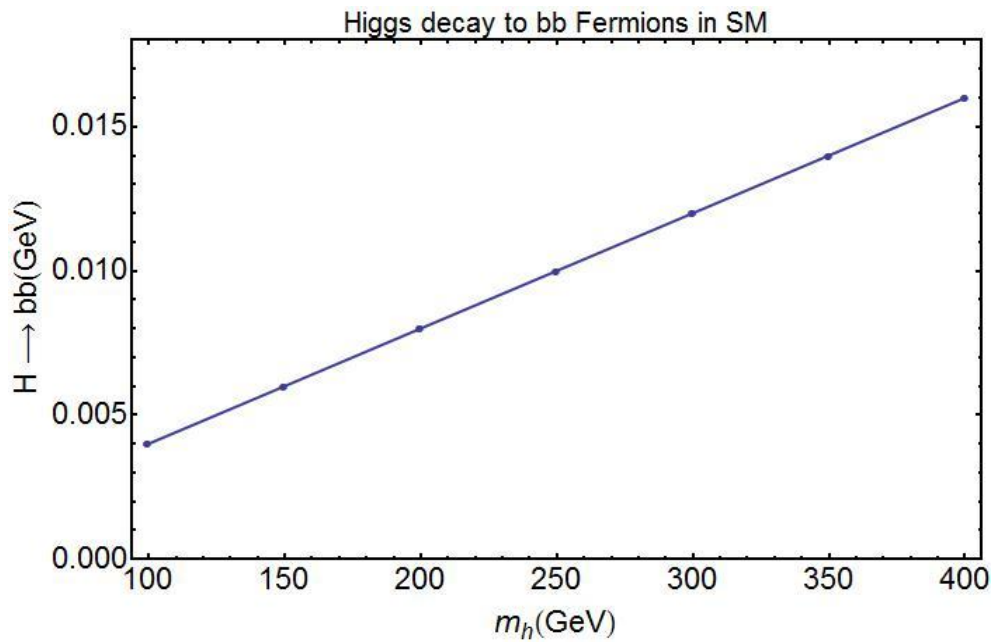
Figure(4-4): Higgs decay to W W boson in the SM as function of its mass.



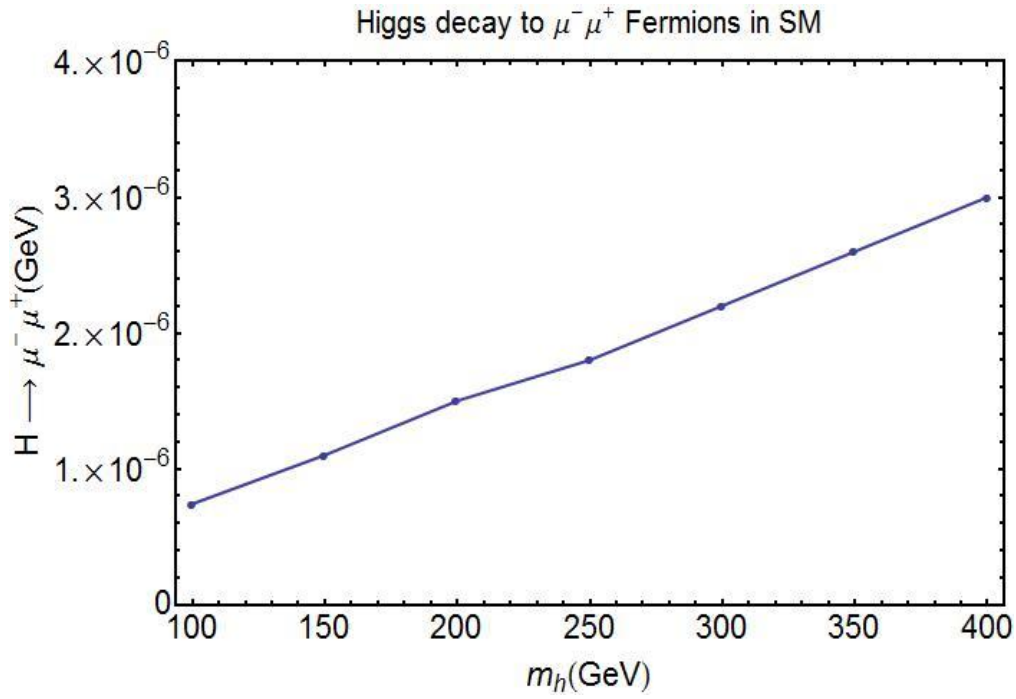
Figure(4-5): Higgs decay to Z Z boson in the SM as function of its mass.



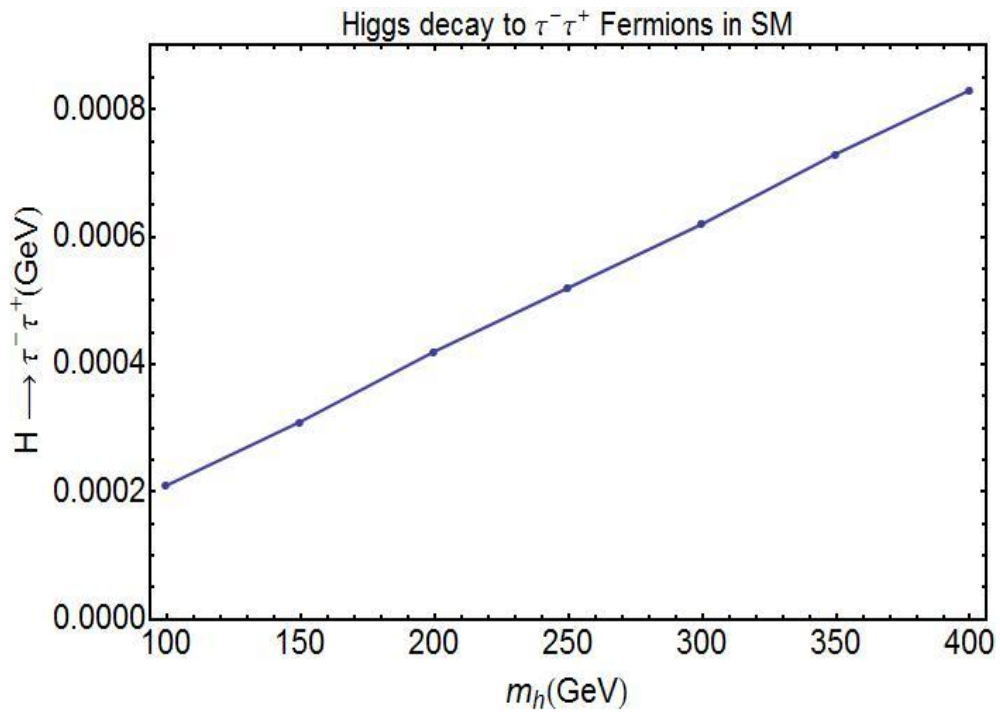
Figure(4-6): Higgs decay to t t quarks in the SM as function of its mass.



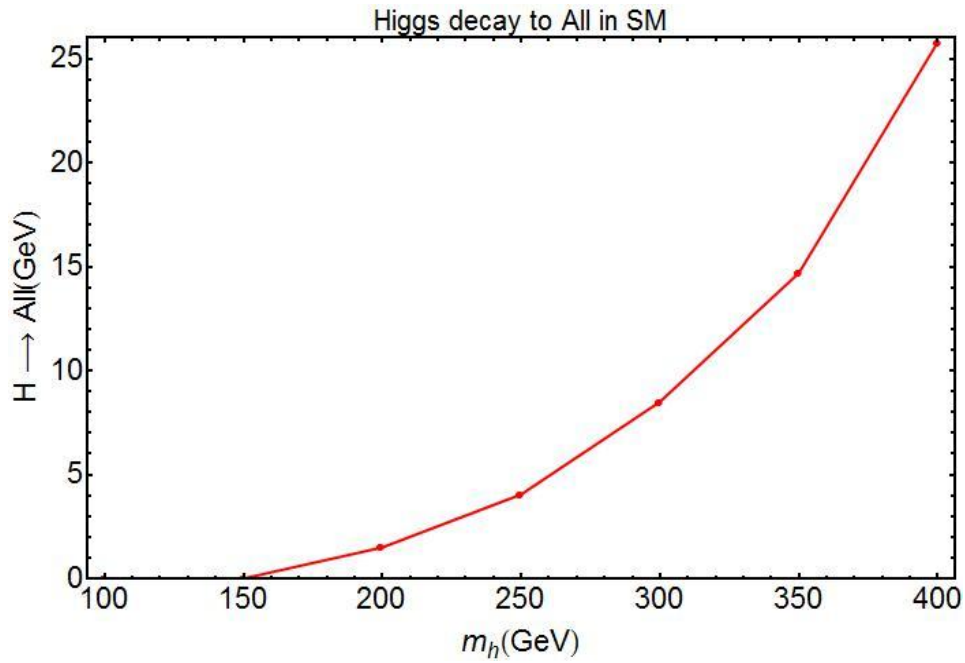
Figure(4-7): Higgs decay to b b quarks in the SM as function of its mass.



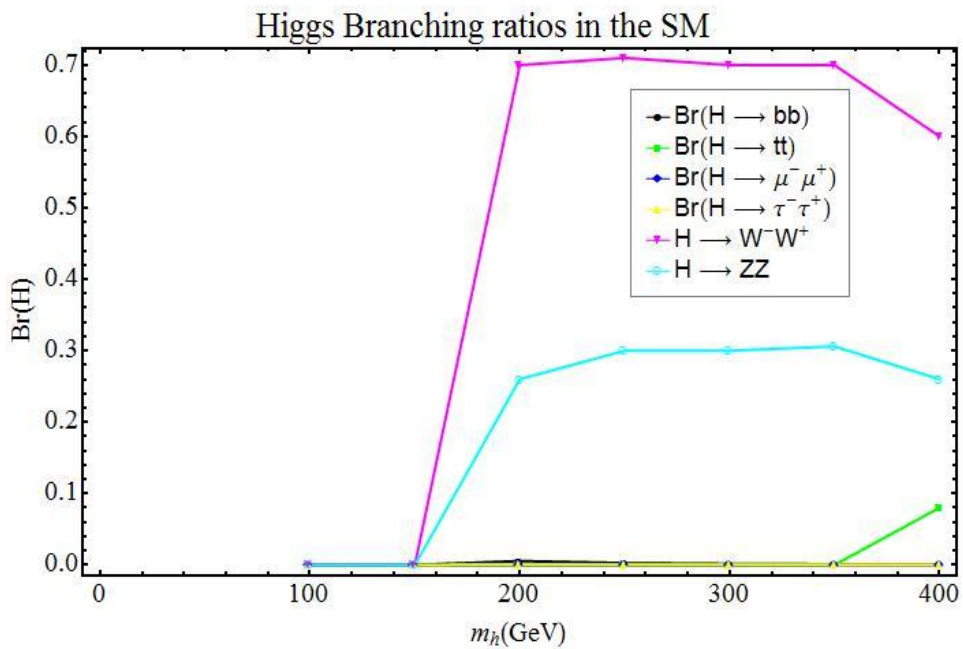
Figure(4.8): Higgs decay to $\mu^- \mu^+$ boson in the SM as function of its mass.



Figure(4.9): Higgs decay to $\tau^- \tau^+$ boson in the SM as function of its mass.



Figure(4-10): Higgs decay to All particles in the SM as function of its mass.



Figure(4-11): Higgs Branching ratios to fermions and bosons.

Chapter Five

Discussion and Conclusion

(5-1) Results and Discussion:

As pictured in Figure (4-1) the Higgs decay rate into W gauge boson tends to increase as the Higgs mass increases. And the decay rate is zero with Higgs mass twice the mass of W gauge boson. We observe similar behavior for the Higgs decaying into Z boson as well as other particles. But in order for the Higgs boson to decay to a given particle its mass should be bigger than twice the mass of the particle as can be seen in Figures conclusion.

By comparing the Higgs decay rates for various channels, we found numerically that the Higgs decay to tau lepton is 10 smaller than the Higgs decaying to bottom quark $H \rightarrow \tau\tau \ll 10 H \rightarrow bb$; such effect can be safely ignored in our calculation.

In particle physics and nuclear physics, the **branching ratio** for decay rate is the fraction of particles which decay by an individual decay mode with respect to the total number of particles which decay and this branching ratio can be defined as follow:

$$Br(H) = \frac{\Gamma(H \rightarrow any\ particle)}{\Gamma(H \rightarrow All)} \quad (5.1)$$

The branching ratios of the SM Higgs boson are presented in figure (4-11) The main decay channel so far in the Higgs mass range is $H \rightarrow WW$ with $Br \sim 68\%$ followed by the decay into ZZ with $Br \sim 29\%$.and tt with $Br \sim 0.9\%$.

(5-2) Conclusion:

In this thesis we studied and discussed the standard model of particle physics in details. Moreover we discussed the Higgs mechanism at length, then we calculated the decay rates of the Higgs in the Standard Model of particle physics, as well as the branching ratios of the Higgs gauge boson in details at tree level. We find that the dominant decay channel is the Higgs decaying into WW gauge boson with $Br \sim 68\%$ this result hold for $m_h \geq 142 \text{ GeV}$.

The total decays widths of the Higgs bosons and the various branching ratios in the SM are discussed.

This work may be extended by including the effects from the loop contribution such as Higgs decaying into two photon or two gluon fusion which appear only at loop level.

(5-3) Recommendation:

We must explain how one can see the Higgs, hence it is unstable; it will be seen through its decays products. We demonstrated by our numerical calculations, that the decay of the Higgs into $t\bar{t}$. However, for heavy Higgs, $m > \text{TeV}$ one gets $\Gamma_{(H \rightarrow VV)}$ rather large.

Obviously for such heavy Higgs there is no sense of saying an elementary Higgs, and (λ) self coupling for the Higgs potential:

$$V = \frac{\lambda}{4} (\Phi^\dagger \Phi - \Phi_0^2)^2 \quad (5.2)$$

Becomes non-perturbative and we cannot trust our computations.

Therefore, the great success of the perturbative high precision, the standard model indicate strongly that the Higgs must be much lighter than TeV, to be found at the LHC.

The $H \rightarrow ZZ$ decay is partially important since one identifies $Z \rightarrow \tau\tau$ pairs without any loss of energy.

Actually $H \rightarrow \gamma\gamma$ becomes more useful, the Higgs decay into two photons must be produced through loops such as

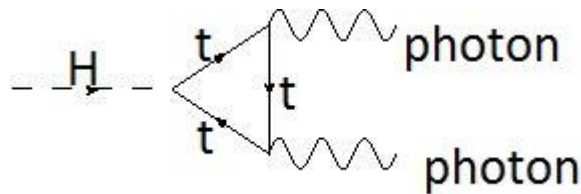


Figure (5-1): Higgs decay to photon.

Where the top quark is selected in the fermionic loop due to its large Yukawa coupling for the $m_h < 140$ GeV; it will be the best way to search for the Higgs, then you can imagine in this range the Higgs discovery may take along time, perhaps some years due to the background and the weakness of the process at loop level. It will be much easier to see the heavy Higgs, $m_h > 180$ GeV through the ZZ decay. Moreover Higgs also can decay to two gluons through loop as

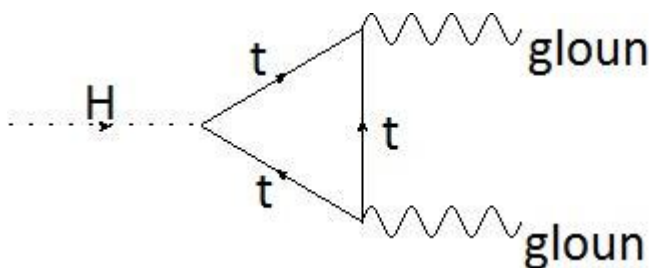


Figure (5-2): Higgs decay to gluon.

Where the top quark again selected inside the loop because of its large Yukawa coupling.

In spite of the loop suppression feature, the gluon fusion turns out to be the dominant Higgs production mechanism at the LHC for all values of the Higgs masses.

The effect of higher order corrections can give significant contribution to the Higgs decay rate, Therefore, it would be appreciated if one can calculate the Higgs decays at one- loop or more than one- loop.

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