

# Chapter One

## Introduction

### **Chapter 1**

#### **Introduction**

#### **(1.1) History of superconductors :**

Superconductors (SCS) are materials that have zero resistance and infinite conductivity below a certain critical temperature . Superconductivity (SC) was discovered in 1911 in the Leiden laboratory at Holland , where the resistivity of Hg vanished about 4K and well below it . Bardeen and Cooper proposed a theoretical model describing a conduction mechanism for super conductors in 1957. [1,2,3]

Till 1986 , critical temperatures at which resistance disappears were always less than 23K. In 1986, Bednarz and Mueller published a paper , showing new materials , having conductivity of about 135K.They a warded Nobel prize at 1987. These new class of materials are known as high temperature superconductors (HTSCS). [4,5,6,7]

Superconductors are widely used in many applications. In medicine ,power ful superconducting magnets (SCM) are used in magnetic resonance imaging (MRI) devices for diagnosis . superconductors are also used to generate powerful electric energy from powerful magnetic field . high speed trains also utilizes SCS to generate powerful magnetic field . [8,9,10]

#### (1.2) **Research problem :**

The theoretical frame wave of SCS are not yet well established.[11]. There is no single simple model that describes SC . [12,13]

#### (1.3) **Literature review :**

Different attempts were made to describe SC using simple models [14]. In some of them plasma equation is used to find temperature dependent Schrödinger equation .

This equation is used to find critical temperature of HTSC [15]. Some models are based on Maxwell's equation for resistive medium [16]. This model can describe SCS .

#### (1.4) **Aim of the work :**

The aim of this work is to develop simple quantum model to describe Josephson effect .

(1.5) **Presentation of the thesis :**

The thesis consists of three chapters . chapter one is the introduction , chapter two is concerned with SCS phenomenon , while chapter three is devoted for the contribution which is concerned with derivation of new Josephson conductivity equation .

# Chapter two

## Superconductivity

## **Chapter 2**

### **Superconductivity**

#### **2-1 Introduction :**

Superconductors have been studied intensively for their fundamental interest and for their useful technological applications.

Until 1986, critical temperatures ( $T_c$ 's) at which resistance disappears were always less than about 23K. In 1986, Bednorz and Mueller discover a new class of materials which currently include members with  $T_c$ 's of about 135K.

This chapter is concerned with superconductors properties as well as electrical and magnetic properties .

#### **2-2 Properties of superconductors:**

Superconducting materials exhibit unusual behaviors like zero resistance below a material critical temperature  $T_c$  , the DC electrical resistance is really zero, not just very small.

If a current is set up in a superconductor with a torus, it will flow forever without any driving voltage. (In practice, experiments have been performed in which persistent currents flow for several years without signs of degrading). A superconductor also expels a weak magnetic field nearly

completely from its interior (screening currents flow to compensate the field within a surface layer of a few 100 or 1000 Å, and the field at the sample surface drops to zero over this layer). Thermodynamic properties of superconductors indicate the existence of an energy gap. When there is a gap, only small number of particles have enough thermal energy to be promoted to the available unoccupied states above the gap.

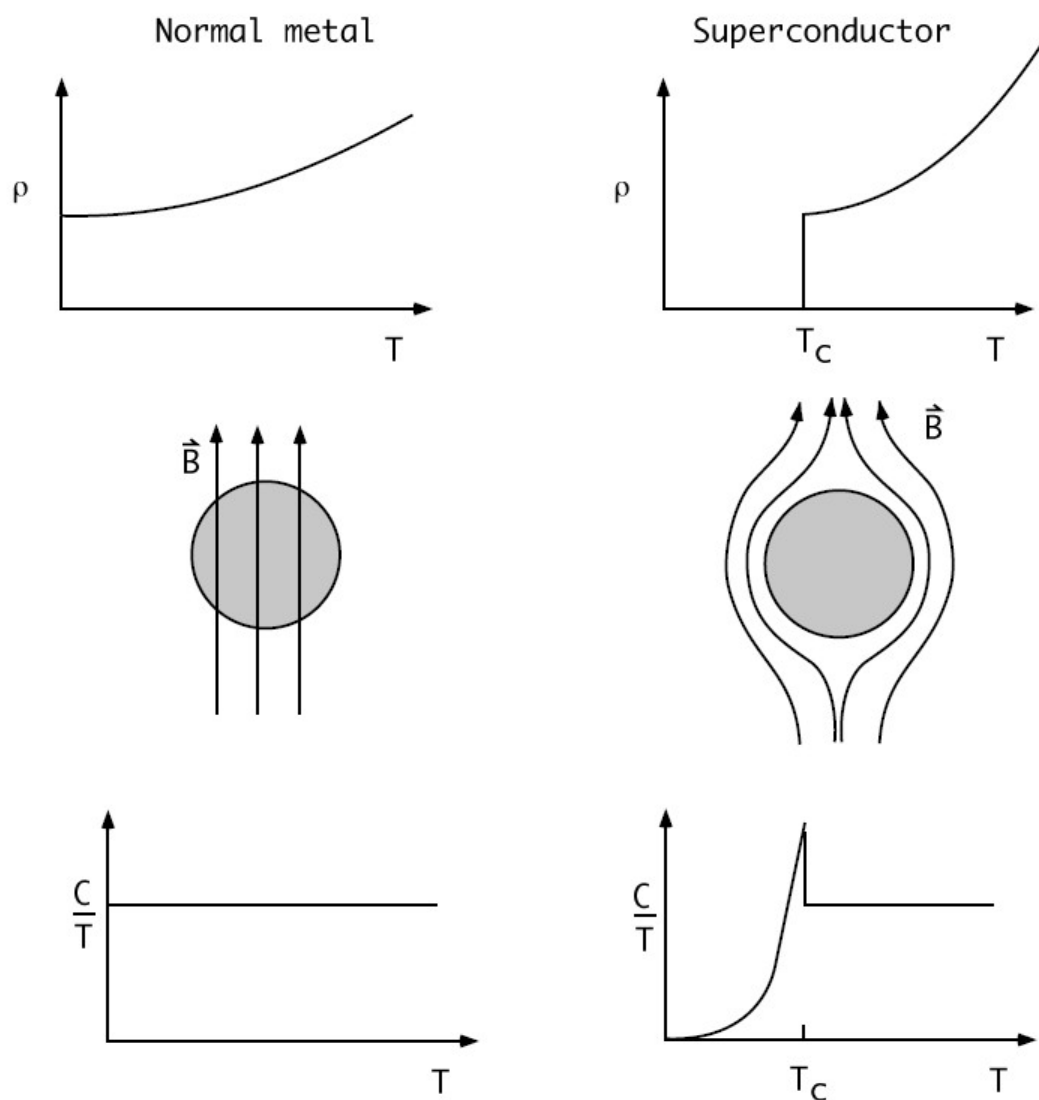


Figure 1: Properties of superconductors.

## **2-3 Electron-phonon interaction as superconducting mechanism:**

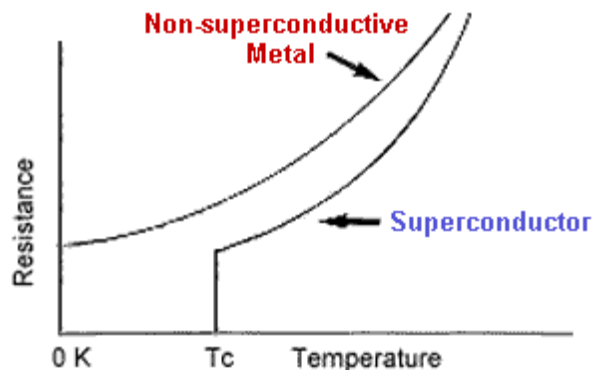
Charge transport in SCS is due to an effective attraction between conduction electrons. Since two electrons experience a repulsive Coulomb force, there must be an additional attractive force between two electrons. In classic superconductors, this force is known to arise from the interaction with the ionic system.

For a normal metal, the ions were replaced by a homogeneous positive background which enforces charge neutrality in the system. In reality, this medium is polarizable, the number of ions per unit volume can fluctuate in time. In particular, if we imagine a snapshot of a single electron entering a region of the metal, it will create a net positive charge density near itself by attracting the oppositely charged ions. It is important to note that a typical electron close to the Fermi surface moves with velocity  $v_F$ , which is much larger than the velocity of the ions,  $v_i$ . So by the time ( $\tau \approx 10^{-13}$  sec) the ions have polarized themselves, the first electron is long gone (it's moved a distance  $v_F \tau \approx 10^8$  cm/s  $\approx 1000$  Å), and the second electron can lower its energy within the concentration of positive charge before the ionic fluctuation relaxes away. This gives rise to an effective attraction between the two electrons, which may be large enough to overcome the repulsive Coulomb interaction. Historically, this electron-phonon "pairing" mechanism was suggested by Frölich in 1950, and confirmed by the discovery of the "isotope effect", where the critical temperature  $T_c$  was found to vary as  $v_i^{-1/2}$  for materials which were identical chemically but which were made with different isotopes.

## **2-4 Superconductors Types :**

there are two types of superconductors:

**a. Type- I superconductors:**



The Type- I category of superconductors is mainly comprised of pure metals that normally show some conductivity at room temperature. They require incredible cold to slow down molecular vibrations sufficiently to facilitate unimpeded electron flow in accordance with what is known as bardeen cheever SC theory (BCS).

"Cooper pairs" in order to help each other overcome molecular obstacles -much like race cars on a track drafting each other in order to go faster. Type- I superconductors , also known as the "soft" superconductors , were discovered first and require the coldest temperatures to become superconductive. They are characterized by a very sharp transition to a superconducting state and by "perfect" diamagnetism , the ability to repel a magnetic field.

**b. Type-II superconductor:**

Except for the elements vanadium, technetium and niobium, the Type II superconductor category of superconductors is comprised of metallic compounds and alloys. The recently discovered superconducting "perovskites" (metal-oxide ceramics that normally have a ratio of 2 metal atoms to every 3 oxygen atoms) belong to this Type II superconductor group. They achieve higher T<sub>c</sub>'s than Type I superconductors by a mechanism that is still not completely understood. Conventional wisdom holds that it relates to the planar

layering within the crystalline structure. Although, other recent research suggests the holes of hypocharged oxygen in the charge reservoirs are responsible. The superconducting cuprates (copper-oxides) have achieved astonishingly high  $T_c$ 's, on theory predicts an upper limit of about 200K for the layered cuprates. Others assert there is no limit. Either way, it is almost certain that other, more-synergistic compounds still await discovery among the high-temperature superconductors.[ 1,2,3 ]

W. De Haas and J. Voogd fabricated The first superconducting Type II compound, an alloy of lead and bismuth, in 1930, however, was not recognized as such until much later, after the Meissner effect had been discovered The first of the oxide superconductors was created in 1973 by dupont researcher art sleight when  $Ba (pb, Bi) O_3$  was found to have a  $T_c$  of 13 k .

Type-II superconductor - also known as the "hard" superconductors , differ from Type- I in that their transition from a normal to a superconducting state is gradual across a region of "mixed state" behavior. A Type- II superconductor will also allow some penetration by an external magnetic field into its surface. A Type I will not.[ 4,5,6 ]

## 2-5 Derivation of first London equation:

A potential difference applied along a conducting wire produces an electric field  $E$  , and hence the force  $F$  on any electron is given by:

$$F = -eE = m \frac{dv}{dt},$$

Where  $e$  is the electron charge ,  $m$  represents its mass, while  $v$  stands for its velocity. Electrons undergo successive periods of acceleration interrupted by collision, and during the average time [relaxation time (scattering on defects)]  $\tau$  between collision. The velocity is given by:



$$v = \frac{-eE}{m} \tau$$

(2-1)

Which called the drift velocity, the negative sign means that the electrons move in a direction opposite to that of the electric field.

When the electron is assumed to move in a resistive medium- which have frictional force proportional to the velocity- the electron equation of motion is given by:

$$m \frac{dv}{dt} = eE - m \frac{v}{\tau}$$

(2-2)

Where the frictional force is given by:

$$F = \frac{mv}{\tau}$$

For steady state in normal metal, no acceleration exists. i.e.

$$\frac{dv}{dt} = 0,$$

Therefore

$$v = \frac{eE}{m} \tau$$

(2-3)

Hence the current density given by:

$$J = nev = \frac{ne^2 \tau}{m} E = \sigma E$$

(2-4)

Where  $n$  is the density of electrons  $\sigma$  is electrical conductivity.

In the two-fluid model we have the temperature-dependents expression for the super  $n_s$  and normal  $n_n$  electrons densities respectively.

$$n_s(T) + n_n(T) = n$$

(2-5)

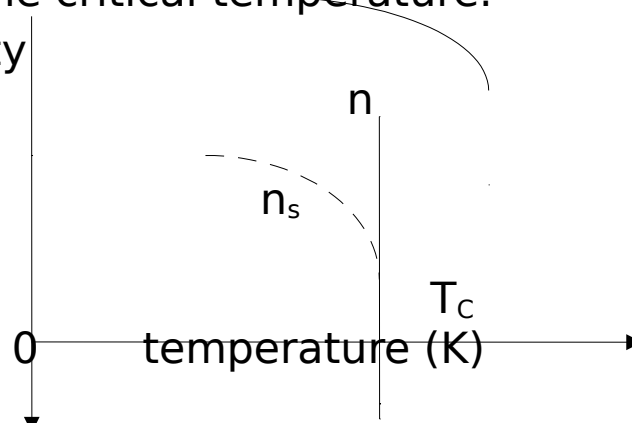
Where the total electron density  $n$  is independent of temperature and at  $T=0$  we have  $n_n(0)=0$  and  $n_s(0)=n$ , and the simple theory predict the following temperature dependence:

$$n_s(T) = n \left( \frac{T}{T_c} \right)^4$$

(2-6)

Where  $T_c$  is the critical temperature.

Electron density



Figer (2-3): temperature dependence of density of superconducting electron  $n_s$ .

For superconductor below.  $T_c$  the resistivity is zero , we obtain equation (2-2) become:

$$\frac{dv}{dt} = \frac{eE}{m}$$

(2-7)

Taking the derivative of  $J$  in equation (2-4) with respect to time:

$$\frac{dJ}{dt} = n_s e \frac{dv}{dt} = \frac{n_s e^2}{m} E$$

(2-8)

The term  $\frac{m}{n_s e^2} = \Lambda$  is a phenomenological parameter.

Equation (2-8) can be rewrite as:

$$E = \frac{d}{dt} \left( \Lambda J \right) = \Lambda \frac{dJ}{dt}$$

(2-9)

This equation is known as the first London equation.

## 2-6 Second London equation:

The equation relates to time-dependent fields, and important for Meissner effect. The electric current density is given quite generally by:

$$J = nqv$$

(2-10)

Where  $n$  is concentration of carriers of charge  $q$ . in the presence of a magnetic field described by the vector potential  $A$ , the velocity  $v$  is related to the total momentum  $p$  by:

$$p = mv + \frac{q}{c} A; v = \frac{1}{m} \left( p - \frac{q}{c} A \right) \quad (2-11)$$

Where  $m$  is the mass,  $c$  the speed of light in vacuum .

Thus equation (2-10) can written as:

$$J = \frac{nq}{m} p - \frac{nq^2}{mc} A$$

(2-12)

In the superconducting state, the total momentum  $p$  is zero, although it not equal to zero in normal state. i.e.

$P=0$ , And equation (2-12) reduces to:

$$J = \frac{-nq^2}{mc} A$$

(2-13)

For electrons,  $q=e$ ,  $n=n_s$

Then:

$$J = \frac{-n_s e^2}{mc} A$$

(2-14)

The vector potential is related to the magnetic field by:

$$B = \nabla \times A.$$

(2-15)

Equation (2-14) can be rewritten as:

$$J = \frac{-c}{4\pi \lambda_L^2} A$$

(2-16)

The equation is known as the second London equation.

Where  $\lambda_L^2 = \frac{mc^2}{4\pi n_s e^2}$  (where  $\lambda_L$  is known as London penetration depth) .[2]

## 2-7 Josephson superconductor tunneling effects:

Consider two metals separated by an insulator, as in the figure below :

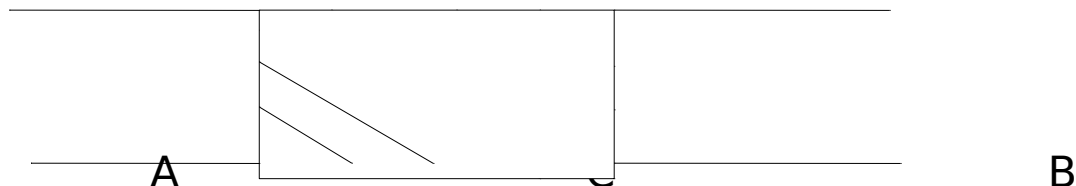
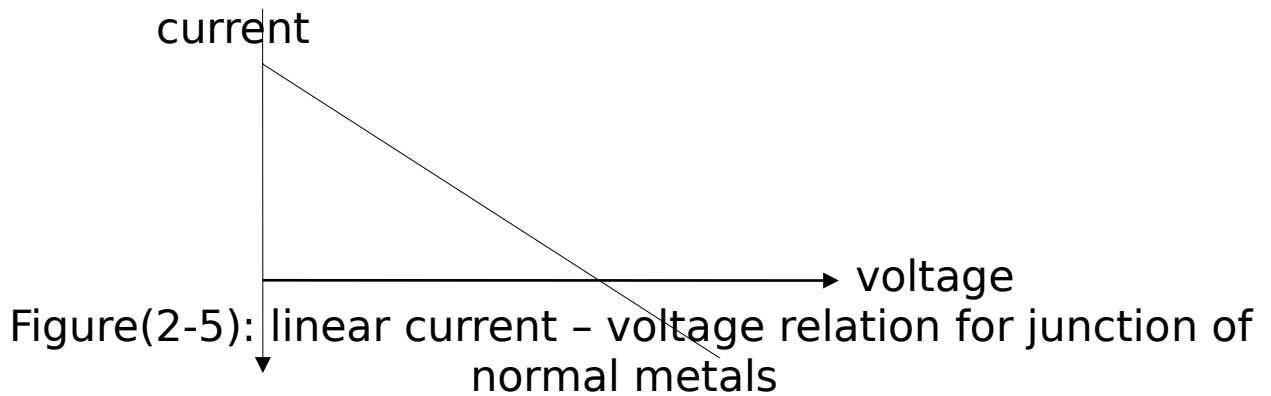


Figure (2-4): two metals A and B separated by a thin layer of insulator C .

the insulator normally acts as a barrier to the flow of conduction electrons from one metal to the other. If the barrier is sufficiently thin (less than 10 or 20 Å), there is a significant probability that an electron, which impinges on the barrier, will pass from one metal to the other: this is called tunneling. The concept that particles can tunnel through potential barriers is as old as quantum mechanics .

When both metals are normal conductors, the current - voltage relation of sandwich or tunneling junction is ohmic at low voltage , with the current directly proportional to the

applied voltage . Giaever discovered that if one of the metals becomes superconducting the current – voltage characteristic changes from the straight line to curve .



In the superconductors there is an energy gap centered at the Fermi level at absolute zero no current can flow until the voltage is  $V = E_g/2e = \Delta/e$ , where  $\Delta = E_g/2$  .

The energy gap  $E_g$  corresponds to the break-up of a pair of electrons in the superconducting state , with the formation of two electrons, or an electron and a hole, in the normal state .

The current starts when  $eV = \Delta$ . At temperatures different from zero there is a small current flow even at low voltage ,

because of electrons in the superconductor that are thermally excited across the energy gap.

Under suitable conditions, remarkable effect can be observed associated with the tunneling of superconducting electron pairs from a superconductor through a layer of an insulator in to another superconductor. The effects of pair tunneling are quite unlike single particle tunneling and include:

I. **DC Josephson effect :**

In this effect A dc current flows across the junction in the absence of any electric or magnetic field.

II. **AC Josephson effect :**

In this effect A dc voltage applied across the junction causes radio frequency (RF) current oscillations across the junction. This effect has been utilized in a precision determination of the value of  $\frac{h}{2e}$ . Further, an ( RF) voltage applied with the dc voltage can then cause a dc current across the junction.

$\frac{h}{2e}$

III. **Macroscopic long-range quantum interference:**

A dc magnetic field applied through a superconducting circuit containing two junctions causes the maximum super current to show interference effect as a function of magnetic field intensity. This effect can be utilized in sensitive Magnetometers.

I. **DC Josephson Effect:**

Let  $\psi_1$  be the probability amplitude of electron pairs on one side of a junction and  $\psi_2$  be the amplitude on the other side. For the simplicity, let both superconductors be identical. for the present, we suppose that they are both at zero potential.

The time- dependent Schrödinger equation  $i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$  applied to the two amplitudes gives:

$$i\hbar \frac{\partial \Psi_1}{\partial t} = \hbar T \Psi_2, \quad i\hbar \frac{\partial \Psi_2}{\partial t} = \hbar T \Psi_1, \quad (2-17)$$

Here  $\hbar T$  represents the effect of the electron- pair coupling or transfer interaction across the insulator; Thus the dimensions of a rate or frequency. It is a measure of the leakage of  $\Psi_1$  into the region 2 , and  $\Psi_2$  into the region 1. If the insulator is very thick ,T is zero and there is no pair tunneling .

$$\text{Let } \Psi_1 = \frac{1}{n_1^{\frac{1}{2}}} e^{i\theta_1}, \quad \Psi_2 = \frac{1}{n_2^{\frac{1}{2}}} e^{i\theta_2} \quad (2-18)$$

Where  $n_1$  is the electron density in the region 1 ,  $\theta_1$  is the phase angle ,  $n_2$  is the electron density in the region 2 ,  $\theta_2$  is the phase angle .

Then:

$$\frac{\partial \Psi_1}{\partial t} = \frac{1}{2} n_1^{-\frac{1}{2}} e^{i\theta_1} \frac{\partial n_1}{\partial t} + i \Psi_1 \frac{\partial \theta_1}{\partial t} = iT \Psi_2 \quad (2-19)$$

With the use of equation (2-17) in the form  $\frac{\partial \Psi_1}{\partial t} = iT \Psi_2$  similary,

$$\frac{\partial \Psi_2}{\partial t} = \frac{1}{2} n_2^{-\frac{1}{2}} e^{i\theta_2} \frac{\partial n_2}{\partial t} + i \Psi_2 \frac{\partial \theta_2}{\partial t} = -iT \Psi_1 \quad (2-20)$$

Multiplying equation (2-19) by  $n_1^{\frac{1}{2}} e^{-i\theta_1}$  with  $\delta \equiv \theta_2 - \theta_1$  one obtains:

$$\frac{\partial n_1}{2 \partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = iT \dot{\varphi} n_1 n_2 \dot{\varphi}^{\frac{1}{2}} e^{i\delta} \quad (2-21)$$

Multiplying equation (2-20) by  $n_2^{\frac{1}{2}} e^{-i\theta_2}$  with  $\delta \equiv \theta_2 - \theta_1$  one obtains:

$$\frac{\partial n_2}{2 \partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -iT \dot{\varphi} n_1 n_2 \dot{\varphi}^{\frac{1}{2}} e^{i\delta} \quad (2-22)$$

Now equating the real and the imaginary parts of equation (2-21) and equation (2-22) , similarly one gets:

$$\frac{\partial n_1}{\partial t} = 2T \dot{\varphi} n_1 n_2 \dot{\varphi}^{\frac{1}{2}} \sin \delta \quad (2-23)$$

$$\frac{\partial \theta_1}{\partial t} = -T \dot{\varphi} \frac{n_2}{n_1} \dot{\varphi}^{\frac{1}{2}} \cos \delta \quad (2-24)$$

If  $n_1 \approx n_2$  as for identical superconductors 1 and 2 , it follows from equation (2-24) that:

$$\frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_2}{\partial t}, \frac{\partial}{\partial t}(\theta_2 - \theta_1) = 0 \quad (2-25)$$

From equation (2-23) it is clear that:



$$\frac{\partial n_2}{\partial t} = -\frac{\partial n_1}{\partial t}$$

(2-26)

The current flow from 1 to 2 is proportional to  $\frac{\partial n_2}{\partial t}$ , the same thing from 2 to 1 is proportional to  $\frac{-\partial n_1}{\partial t}$ .

Therefore one can conclude from equation (2-23) that the current  $J$  of the superconductor pairs across the junction depends on the phase difference  $\delta$  as

$$J = \frac{\partial n_1}{\partial t} - \frac{\partial n_2}{\partial t} = T \hbar n_1 n_2 \sin \delta$$

$$J = J_0 \sin \delta = J_0 \sin(\theta_2 - \theta_1) \quad (2-27)$$

Where  $J_0$  is proportional to the transfer interaction  $T$ . the current  $J_0$  is the maximum zero- voltage current that can be passed through the junction .

With no applied voltage a dc current will flow across the junction , [figure (2-7)] with a value between  $J_0$  and  $-J_0$  according to the value of the phase difference  $\theta_2 - \theta_1$  . This is the dc Josephson effect .

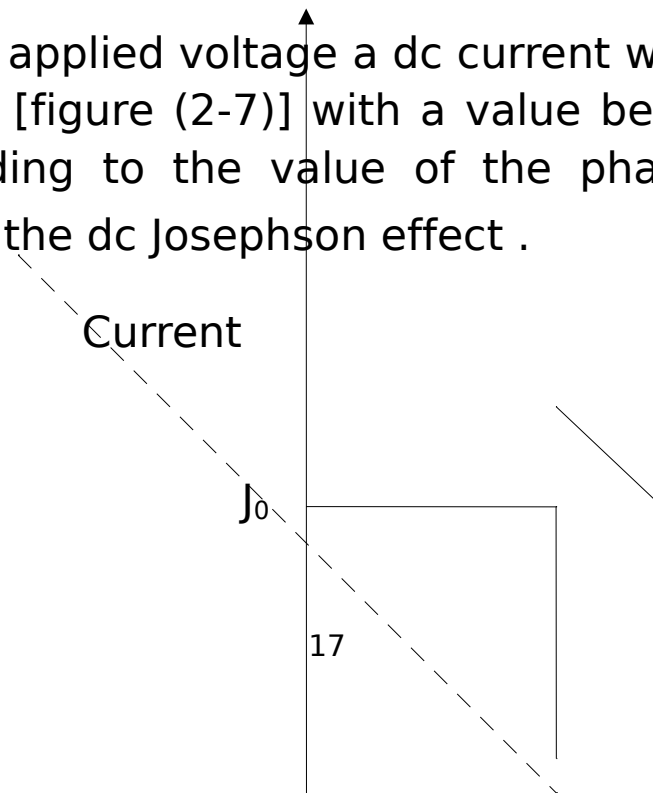




Figure (2-7): current- voltage characteristic of a Josephson junction.

Dc currents flow under zero applied voltage up to a critical current  $i_c$  ; this is the dc Josephson effect. At voltage above  $v_c$  the junction has a finite resistance , but the current has an oscillatory component of frequency  $\omega = 2eV\hbar$  , this ac Josephson effect.

## II. **AC Josephson effect:**

Let a voltage  $V$  be applied across the junction. This can be done because the junction is an insulator. An electron pair experience a potential energy difference  $qV$  on passing across the junction, where  $q=-2e$ . one can say that a pair on one side is at potential energy  $-eV$  and a pair on the other side at  $eV$ .

The equations of motion that replaces (2-17) are:

$$i\hbar \frac{\partial \Psi_1}{\partial t} = \hbar T \Psi_2 - eV \Psi_1, i\hbar \frac{\partial \Psi_2}{\partial t} = \hbar T \Psi_1 - eV \Psi_2 \quad (2-28)$$

Proceeding as above to find in place of (2-21) the equation:

$$\frac{1}{2} \frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = \frac{i e n_1 V}{\hbar} - i T \quad (2-29)$$

Taking the real parts on both sides one gets:

$$\frac{\partial n_1}{\partial t} = 2T \sin \delta \quad (2-30)$$

The imaginary contribution also gives:

$$\frac{\partial \theta_1}{\partial t} = \frac{eV}{\hbar} - T \cos \delta \quad (2-31)$$

Which differs from equation (2-24) by the term  $\frac{eV}{\hbar}$ .

Similarly, as equation (2-29) the equation (2-22) for  $n_2$  takes the form:

$$\frac{1}{2} \frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -i \frac{eV n_2}{\hbar} - i T \quad (2-32)$$

Hence, equating real and imaginary parts one gets:

$$\frac{\partial n_2}{\partial t} = -2T \sin \delta \quad (2-33)$$

$$\frac{n_1}{n_2} i^{\frac{1}{2}} \cos \delta$$

$$\frac{\partial \theta_2}{\partial t} = \frac{-eV}{\hbar} - T i$$
(2-34)

From (2-31) and (2-34) with  $n_1 \approx n_2$  one have:

$$\frac{\partial(\theta_2 - \theta_1)}{\partial t} = \frac{\partial \delta}{\partial t} = \frac{-2eV}{\hbar}$$

(2-35)

By integration of (2-35) that with a dc voltage across the junction the relative phase of the probability amplitudes vary as:

$$\delta(t) = \delta(0) - \frac{2eV}{\hbar} t$$

(2-36)

The current is now given by (2-27) and (2-36) to be:

$$J = J_0 \sin \left[ \delta(0) - \frac{2eV}{\hbar} t \right]$$

(2-37)

The current oscillates with frequency:

$$\omega = \frac{2eV}{\hbar}$$

(2-38)

Which says that a photon of energy  $\hbar\omega = 2eV$  is emitted or absorbed when an electron pair crosses the barrier . By measuring the voltage and the frequency, it is possible to

obtain a very precise value of  $\frac{e}{\hbar}$  .[8,9,13]

# Chapter three

## New Derivation of Simple Josephson Effect Relation

# Using New Quantum Mechanical Equation

## Chapter 3

### **New Derivation of Simple Josephson Effect Relation Using New Quantum Mechanical Equation**

#### **(3.1) Introduction:**

The Josephson effect relation derivation in standard texts is complex. Thus one needs a simple derivation. This is done by deriving first a new quantum equation. Then solving this equation to get Josephson relation.

#### **(3.2) New Quantum Equation:**

The Newtonian energy  $E$  is a sum of kinetic and potential energy  $v$ , i.e:

$$E = \frac{1}{2} m v^2 + V = \frac{P^2}{2m} + V \quad (3-1)$$

Where  $m$ ,  $v$ ,  $p$  are the mass velocity and momentum respectively. For very small momentum and very large potential one can neglect the first term to get:

$$E = V \quad (3-2)$$

Squaring both sides yields:

$$E^2 = V^2 \quad (3-3)$$

Multiplying both sides by  $\Psi$ , one gets:

$$E^2 \Psi = V^2 \Psi \quad (3-4)$$

The wave function of a free particle is given by:

$$\Psi = A e^{\frac{i}{\hbar}(P_X - Et)} \quad (3-5)$$

Differentiating both sides with respect  $x$  and  $t$  twice

$$\frac{\partial \Psi}{\partial t} = \frac{-i}{\hbar} E \Psi$$

$$\frac{\partial^2 \Psi}{\partial t^2} = \frac{-i}{\hbar} E \frac{\partial \Psi}{\partial t} = \frac{i^2}{\hbar^2} E^2 \Psi = \frac{-E^2}{\hbar^2} \Psi$$

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = E^2 \Psi$$

(3-6)

Similarly:

$$\frac{\partial \Psi}{\partial x} = \frac{i}{\hbar} P \Psi$$

$$\nabla^2 \Psi = \frac{\partial^2 \Psi}{\partial x^2} = \frac{iP}{\hbar} \frac{\partial \Psi}{\partial x} = \frac{iP}{\hbar} \left( \frac{iP}{\hbar} \right) \Psi = \frac{i^2 P^2}{\hbar^2} \Psi$$

$$-\hbar^2 \nabla^2 \Psi = P^2 \Psi$$

(3-7)

Substitute (3-6) in (3-4):

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = V^2 \Psi$$

(3-8)

### **(3-3) Josephson Effect equation:**

To derive Josephson effect equation consider the solution

$$\Psi = D \sin(\alpha t + \phi) \quad (3-9)$$

The potential is constant inside superconductor, thus

$$V = V_0 \quad (3-10)$$

From (3-9)

$$\frac{\partial \Psi}{\partial t} = \alpha D \cos(\alpha t + \phi)$$



$$\begin{aligned}\frac{\partial^2 \Psi}{\partial t^2} &= -\alpha^2 D \sin(\alpha t + \phi) \\ &= -\alpha^2 \Psi\end{aligned}\tag{3-11}$$

Substitute (3-10) and (3-11) in (3-8) one gets:

$$+\hbar^2 \alpha^2 \Psi = V_0^2 \Psi$$

$$\alpha^2 = \frac{V_0^2}{\hbar^2}$$

$$\alpha = \pm \frac{V_0}{\hbar}$$

$$(3-12)$$

Substitute (3-12) in (3-9) and choosing minus sign, one get

$$\Psi = D \sin\left(\frac{-e V_0}{\hbar} t + \phi\right)\tag{3-13}$$

But the energy density J is given by:

$$\begin{aligned}J &= e \frac{\partial n}{\partial t} = e \frac{\partial |\Psi|^2}{\partial t} = 2e |\Psi| \frac{d|\Psi|}{dt} \\ &= 2e D \sin(\alpha t + \phi) \left( \frac{-e}{\hbar} V_0 \right) \cos(\alpha t + \phi) \\ &= -2 \frac{e^2 D V_0}{\hbar} \sin \theta \cos \theta\end{aligned}\tag{3-14}$$

$$\theta = \phi - \frac{eV_0 t}{\hbar}$$

But  $\sin 2\theta = 2\sin\theta \cos\theta$

$$\begin{aligned} & 2\phi - \frac{2eV_0 t}{\hbar} \\ & \quad \quad \quad (\delta) \\ J &= \frac{-e^2 DV_0}{\hbar} \sin \delta \\ &= A \sin\left(2\phi - \frac{2eV_0 t}{\hbar}\right) \end{aligned}$$

(3-15)

Setting:

$$2\phi = \delta(0)$$

The current density is given by:

$$J = J_0 \sin\left(\delta(0) - \frac{2eV_0}{\hbar} t\right) \quad (3-16)$$

Which is  $J_s$  effect equation.

### **(3-4) Discussion :**

Equation (3-2) shows new energy equation based on Newtonian mechanics , with neglected kinetic term . this equation is used to derive a new quantum equation (3-8) . this new equation is based on Newtonian energy with no kinetic term beside the wave equation of a free particle .

This equation is used to derive simple Josephson current density equation . this equation (3-16) is the same as the old one ,but derived using simple arguments .

### **(3-5) conclusion :**

Neglecting kinetic Newtonian term in the energy expression , one can easily derive new quantum equation . this equations ,is shown, to be successful ,in deriving simple Josephson current density equation .

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