Monte Carlo Simulation for Boron Carbide as Effective Control Rods in Thermal Reactors

By:
Rami Abdelmoti Elneem Ahmed

Thesis submitted to the college of graduate studies in fulfillment of the requirements of the degree of M.Sc. in Physics

Supervisor:
Dr. Nassreldeen Abdelrazig Abdelbari Elsheikh

December 2014
Dedication

For my Mother

My father

My brothers

My sisters

My teachers

My friends
Acknowledgment

The thanking firstly and finally for allah for completing this work, My deepest gratitude to my promoter Dr. Nassreldeen Abdelrazig Abdelbari Elsheikh, for his guidance and observations.
Abstract

In this study utilized nuclear core –like environment of real nuclear core by Monte Carlo simulator MCNP5. The object of this study is determine the optimal moderator for fast neutron which act to slow down for fast neutron to thermal region, and determine the optimal absorber (control material) for neutron to control for nuclear chain reaction in the reactor.

Utilize the light water H$_2$O and heavy water D$_2$O as moderator materials in cubic shape 90 cm$^3$ and around the moderator there is reflector carbon $^{14}$C to prevents the leakage for neutrons outside the nuclear core, its thickness equal 10cm$^3$, then after select the optimal moderator will use with absorber materials to examine the optimal absorber, utilize control rods which length is 80cm and diameter is 1cm, Those materials are cadmium (Cd), silver (Ag), &Boron carbide (BC$_4$).

After study appears the optimal moderator for fast neutron is heavy water (D$_2$O) than light water H$_2$O. And found the optimal absorber than others.
الخلاصة

في هذه الدراسة تم استخدام قلب مفاعل نووي يشب هيئة المفاعل النووي الحقيقي باستخدام محركي المونتي كارلو، MCNP5. الهدف من هذه الدراسة هو تحديد أمثل مبادئ النيوترونات السريعة من بث من طاقة النيوترونات السريعة إلى النطاق الحراري، وتحديد أمثل مبادئ النيوترونات للتحكم في التفاعل النووي المستقبلي للمفاعل.

تم استخدام كل من الماء الخفيف $D_2O$ والماء التقليد $H_2O$ في شكل مكعب قدرها 90cm$^3$ كمواد مهيئة بدون إستخدام أقطاب تحكم يوجد حوله عاكس للنيوترونات لمنع النيوترونات من التسبب خارج قلب المفاعل الذي سمكه 10cm$^3$، بعد أن تم تحديد أمثل مبادئ تم استخدامه كمهدئ مع مواد الامتصاص (مواد التحكم)، و استخدم أقطاب التحكم في الدراسة بطول 80cm وقطر 1cm، و موادى: cadmiyum، الفضة، Ag، Cd، البورون كربيد $BC_4$.

وبعد الدراسة أضحى أن الماء الخفيف $D_2O$ أمثل مبادئ النيوترونات السريعة من الماء الخفيف. ووجد أن أمثل مبادئ النيوترونات هو البورون كربيد $BC_4$ بعد إجراء المقارنة بينها.
Chapter One

Introduction

1.1 Introduction 1

1.2 Working principle of thermal reactor 1

1.3 Statement of the problem 2

1.4 Methodology 2

1.4.1 Monte Carlo Simulation 2

1.5 Hypothesis of the Research 3

1.6 Objective of The research 3
<table>
<thead>
<tr>
<th>1.7</th>
<th>Significance of the Research</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>Limitations of the Study</td>
<td>4</td>
</tr>
</tbody>
</table>

**Chapter Two**

**Fast Neutron Moderation & Control Rods**

<table>
<thead>
<tr>
<th>2.1</th>
<th>Moderation for Fast Neutron</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>Interaction of Neutron</td>
<td>5</td>
</tr>
<tr>
<td>2.2.1</td>
<td>The Cross-Section Concept</td>
<td>5</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Microscopic cross section</td>
<td>6</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Macroscopic cross section</td>
<td>7</td>
</tr>
<tr>
<td>2.3</td>
<td>Type of Neutron Interaction</td>
<td>8</td>
</tr>
<tr>
<td>2.4</td>
<td>Scattering</td>
<td>9</td>
</tr>
<tr>
<td>2.4.1</td>
<td>Elastic Scattering</td>
<td>10</td>
</tr>
<tr>
<td>2.4.2</td>
<td>Inelastic Scattering</td>
<td>11</td>
</tr>
<tr>
<td>2.5</td>
<td>Nuclear Fission</td>
<td>11</td>
</tr>
<tr>
<td>2.6</td>
<td>Principle of chain reactions</td>
<td>13</td>
</tr>
<tr>
<td>2.7</td>
<td>Chain Reactions in nuclear reactors</td>
<td>14</td>
</tr>
<tr>
<td>2.8</td>
<td>neutron flux</td>
<td>16</td>
</tr>
<tr>
<td>2.9</td>
<td>Mechanism of Fast Neutron Moderation &amp; Thermalization</td>
<td>16</td>
</tr>
</tbody>
</table>
Chapter Three
Monte Carlo Modeling

3.1 Monte Carlo code

3.1.1 Structure of the MCNP5 Input File

3.1.2 Geometry Specifications

3.2 The Proposed Source

3.3 Model of the Control System

3.5 Materials Used

3.3 The Proposed Source

3.3.1 Neutron Source

3.3.2 Model of the Control System

3.4 Materials Used

Chapter Four
Result and Discussion

4.1 Results for (H2O) & (D2O) as moderators without Control
<table>
<thead>
<tr>
<th>Rods</th>
<th>Results for (D2O) as moderator with cadmium (Cd) as control Rod</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>Heavy water (D2O) as moderator with silver (Ag) as control Rod</td>
<td>33</td>
</tr>
<tr>
<td>4.3</td>
<td>Results for (D2O) as moderator with Boron carbide (BC4) as control Rod</td>
<td>34</td>
</tr>
<tr>
<td>4.4</td>
<td>comparisons between Result’s Control Materials</td>
<td>36</td>
</tr>
<tr>
<td>4.5</td>
<td>Conclusion</td>
<td>37</td>
</tr>
<tr>
<td>4.6</td>
<td>Recommendations</td>
<td>37</td>
</tr>
<tr>
<td>4.7</td>
<td>References</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>Appendix</td>
<td>40</td>
</tr>
</tbody>
</table>
List of Figures

<table>
<thead>
<tr>
<th>No</th>
<th>Figure</th>
<th>Page No</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Type of Neutron interaction</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>Thermal nuclear fission $^{235}\text{U}$</td>
<td>13</td>
</tr>
<tr>
<td>2.3</td>
<td>U-235 undergoing a chain reaction in the reactor</td>
<td>15</td>
</tr>
<tr>
<td>3.1</td>
<td>Simulated Neutron Energy Spectrum- 252Cf</td>
<td>26</td>
</tr>
<tr>
<td>3.2</td>
<td>Model of the Control system without control rods as modeled in MCNP simulations</td>
<td>28</td>
</tr>
<tr>
<td>3.3</td>
<td>Model of the proposed control system with control rods as modeled in MCNP simulations</td>
<td>29</td>
</tr>
<tr>
<td>4.1</td>
<td>Neutron flux as a function of neutron energy in the presence of light water (H2O) &amp; Heavy water (D2O)</td>
<td>31</td>
</tr>
<tr>
<td>4.2</td>
<td>Spectrum of neutron after interact with D2O + Cd</td>
<td>33</td>
</tr>
<tr>
<td>4.3</td>
<td>Spectrum of neutron after interact with D2O + Ag</td>
<td>34</td>
</tr>
<tr>
<td>4.4</td>
<td>Spectrum of neutron after interact with D2O + BC$_4$</td>
<td>35</td>
</tr>
<tr>
<td>4.5</td>
<td>comparison between control materials</td>
<td>36</td>
</tr>
</tbody>
</table>
## List of Tables

<table>
<thead>
<tr>
<th>No</th>
<th>Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Average number of collisions required to reduce a neutron’s energy from 2MeV to 0.025 eV by elastic scattering</td>
</tr>
<tr>
<td>2.2</td>
<td>Moderating powers and ratios of selected materials</td>
</tr>
<tr>
<td>2.3</td>
<td>Comparison of the main characteristics of the most common moderator materials</td>
</tr>
<tr>
<td>2.4</td>
<td>Comparison of the main moderator materials</td>
</tr>
<tr>
<td>3.1</td>
<td>Basic information for $^{252}$Cf Source</td>
</tr>
<tr>
<td>3.2</td>
<td>Properties of materials moderator</td>
</tr>
<tr>
<td>3.3</td>
<td>Properties of materials absorber</td>
</tr>
</tbody>
</table>
بسم الله الرحمن الرحيم

{ وَاتِقُوا اللَّهَ وَيَعْلَمُكُمُ اللهُ وَاللَّهُ بِكُلِّ شَيْءٍ عَلِيمٍ }

صدق الله العظيم
سورة البقرة 282
Chapter One
Introduction
CHAPTER ONE
INTRODUCTION

1.1 Introduction

Most nuclear reactors use a chain reaction to induce a controlled rate of nuclear fission in fissile material, releasing both energy and free neutrons. A reactor consists of an assembly of nuclear fuel (a reactor core), usually surrounded by a neutron moderator such as regular water, heavy water, graphite, or zirconium hydride, and fitted with mechanisms such as control rods that control the rate of the reaction [1].

1.2 Working principles of thermal reactor

To improve probability of fission and enable a chain reaction, uranium-fueled reactors must include a neutron moderator that interacts with newly produced fast neutrons from fission events to reduce their kinetic energy from several MeV to several eV, making them more likely to induce fission. This is because $^{235}$U is much more likely to undergo fission when struck by one of these thermal neutrons than by a freshly-produced neutron from fission. Neutron moderators are materials that interact weakly with the neutrons but absorb kinetic energy from them. Most moderators rely on either weakly bound hydrogen or a loose crystal structure of another light element such as carbon to transfer kinetic energy from the fast-moving neutrons [1]. A moderator must have a high scattering cross section to provide a high collision number of neutrons per time unit prior to being captured, as well as a low neutron-absorption cross section that not to deteriorate a neutron balance owing to nonproductive absorption in moderator mass [2].
1.3 Statement of the problem

The problem of the research is to model a control system suitable for thermal reactors and to examine the effectiveness of some moderating materials and absorbers. This is done in a reactor-like environment containing a $^{252}$Cf-fast neutron source. The geometry of the proposed model and simulations were carried out using Monte Carlo computer code. The proposed system is examined and the optimal operating conditions are reported.

1.4 Methodology

In this work, Monte Carlo computer code (MCNP5), was used to simulate the performance of several materials including; moderators, with which fast neutrons interact to achieve neutrons in thermal range, absorbers in the form of control rods to explore their effectiveness in reducing thermal neutron intensity and carbon as fast neutron reflector. The MCNP5 code was used to model a suitable geometry for a reactor-like environment and explore the performance of the mentioned materials as an effective control system.

1.4.1 Monte Carlo Simulation

Monte Carlo Simulation software MCNP will be used to simulate the interaction of fast neutron with materials (moderators, & absorber). The data will be used to predict the transfer the fast neutron to thermal neutron & absorb for increase fast neutron.
1.5 Hypothesis of the Research

Scattering events can be subdivided into elastic and inelastic scattering. In elastic scattering the total kinetic energy of the neutron and nucleus is unchanged by the interaction. During the interaction a fraction of the neutron’s kinetic energy is transferred to the nucleus [4]. For effective moderation of neutrons in thermal reactors, one will thus have to choose materials with low \( A \) (mass number) values [5]. Consequently, in this work, light water (H\(_2\)O), Heavy water (D\(_2\)O), are presented as moderator materials for fast neutron.

Control rods are used in nuclear reactors to control the fission rate of uranium and plutonium. They are composed of chemical elements such as boron, silver, indium and cadmium that are capable of absorbing many neutrons without fissioning themselves [18]. One will thus have to choose materials of high absorption cross sections are used to reduce the intensity of thermal neutrons and thus control the fission reaction;

\[
^{235}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{142}\text{Ba} + ^{91}\text{Kr} + 3^{1}_{0}\text{n}
\]

1.6 Objective of The research

This work reviews the basic physical mechanisms of the interaction of neutron (fast neutron) with some materials (moderators & absorbers) to model a control system in a reactor-like environment.

1.7 Significance of the Research

The significance of the current study lies on providing physical insight for neutron scattering with moderators and neutron absorption with absorbers in a reactor-like environment. Furthermore, the work provides a suitable geometry for a control system possibly useful in nuclear thermal reactors.
1.8 Limitations of the Study

The research involved in this study, present MCNP simulations for the proposed control system for nuclear thermal reactors and reviews the interaction mechanisms of neutrons with several moderators and absorbers. Further experimental verifications are required and other moderating materials and absorbers should be tested.
Chapter Two

Fast Neutron Moderation & Control Rods
CHAPTER TWO
FAST NEUTRON MODERATION AND CONTROL RODS

2.1 Moderation for Fast Neutron

Inside thermal nuclear reactor, before these neutrons induce additional fissions to sustain chain reaction, their energy is reduced, often by several orders of magnitude, as the result of successive elastic and inelastic collisions with the nuclei in the system. In thermal reactors, for instance, almost all of the fission neutrons slow down to thermal energies before they induce further fissions. By contrast, they interact with the fuel and produce fissions [7]. The cross section for neutron-induced fission is much higher for thermal neutrons (100 meV) than for the fast neutrons (1–2 MeV) that are produced. For a reactor to achieve a self-sustaining chain reaction from a small mass of fissile material, and to obtain suitable neutrons for neutron scattering, the fast neutrons within the core must be “slowed down” [8].

2.2 Interaction of Neutron

2.2.1 The Cross-Section Concept

The probability of a particular reaction occurring between a neutron and a nucleus is expressed through the concept of the cross section. If a large number of neutrons of the same energy are directed into a thin layer of material, some may pass through with no interaction, others may have interactions that change their directions and energies, and still others may fail to emerge from the sample.
There is a probability for each of these events. For example, the probability of a neutron not emerging from a sample (that is, of being absorbed or captured) is the ratio of the number of neutrons that do not emerge to the number originally incident on the layer. The cross section for being absorbed is the probability of neutrons being absorbed divided by the areal atom density (the number of target atoms per unit area of the layer). The cross section thus has the dimensions of area; it must be a small fraction of a square centimeter because of the large number of atoms involved. Because this type of cross section describes the probability of neutron interaction with a single nucleus, it is called the microscopic cross section and is given the symbol $\sigma$ [4].

### 2.2.4 Microscopic cross section Interactions

Another approach to understanding the concept of the microscopic cross section is to consider the probability of a single neutron attempting to pass through a thin layer of material that has an area $A$ and contains $N$ target nuclei, each of cross-sectional areas. The sum of all the areas of the nuclei is $NS$. The probability of a single neutron hitting one of these nuclei is roughly the ratio of the total target area $NS$ to the area of the layer $A$. In other words, the probability of a single neutron having a collision with a nucleus is $NS/A$ or $(N/A)S$, the areal target density times $S$. On the atomic level, however, cross sections for neutron interactions are not simply the geometrical cross sectional area of the target. By replacing this $S$ by the $\sigma$ of the preceding paragraph, $\sigma$ might be thought of as an effective cross-sectional area for the interaction. The cross section for the interaction retains the dimensions of area that $S$ had.
The physical cross-sectional area $s$ of a heavy nucleus is about $2 \times 10^{-24} \text{cm}^2$. Interaction cross sections for most nuclei are typically between $10^{-27}$ and $10^{-21} \text{cm}^2$. To avoid the inconvenience of working with such small numbers, a different unit of area is used the barn, denoted by the symbol b. It is defined to be $10^{-24} \text{cm}^2$, so that the physical cross-sectional area of a heavy nucleus is about 2 b. Many neutron interaction cross sections range between 0.001 and 1000 b.

Each type of event has its own probability and cross section. The probability of each type of event is independent of the probabilities of the other so the total probability of any event occurring is the sum of the individual probabilities. Similarly, the sum of all the individual cross sections is the total cross section [4].

### 2.2.3 Macroscopic cross section

The definition of the macroscopic cross section arises from the transmission of a parallel beam of neutrons through a thick sample. The thick sample can be considered to be a series of atomic layers $\sigma$ for each layer we can apply the results found with the microscopic cross-section concept. By integrating through enough atomic layers to reach a depth $x$ in the sample, the intensity $I(x)$ of the uncollided neutron beam is

$$I(x) = I_0 e^{-N\sigma X} \quad (2.1)$$

Where $I_0$ is the intensity of the beam is before it enters the sample, $N$ this the atom density, $\sigma$ is the the total cross section. Note that the fraction transmitted without collisions $I(X)/I_0$, depends on the energy of the neutrons through the energy dependence of the microscopic total cross-section $\sigma_t$. 
An expression similar to Equation (2.1) is used for gamma-ray attenuation. In that ease, low-Energy gamma rays are very likely to be absorbed and thus removed not only from the parallel beam but from the material entirely. With neutrons at low energies, elastic scattering is the most likely event. Although Equation (2.1) gives the intensity of the neutrons that have had no interaction up to a depth $x$, the actual number of neutrons present that can be detected may be much larger because of multiple scattering multiplication, or finite detector acceptance angle.

### 2.3 Type of Neutron Interaction

A neutron can have many types of interactions with a nucleus. Figure (2.1) shows the types of interactions and their cross sections. Each category of interaction in the figure consists of all those linked below it. The total cross section $\sigma_t$ expresses the probability of any interaction taking place.

A simple notation can be used to give a concise indication of an interaction of interest. If a neutron $n$ impinges on a target nucleus $T$, forming a resultant nucleus $R$ and the release of an outgoing particle $g$, this interaction is shown as $T(n,g)R$. The heavy nuclei are shown outside the parentheses. To denote a type of interaction without regard for the nuclei involved, only the portion in parentheses is shown. An example of an $(n,p)$ reaction is $^7B(n,p)^5Be$.

An interaction may be one of two major types scattering or absorption. When a neutron is scattered by a nucleus, its speed and direction change but the nucleus is left with the same number of protons and neutrons it had before the interaction. The nucleus will have some recoil velocity and it may be left in an excited state that will lead to the eventual release of radiation. When a neutron is absorbed by a nucleus, a wide range of radiations can be emitted or fission can be induced [4].
The types of neutron interactions are summarized in figure (2.1)

2.4 Scattering

A neutron scattering reaction occurs when a nucleus, after having been struck by a neutron, emits a single neutron. Despite the fact that the initial and final neutrons do not need to be (and often are not) the same, the net effect of the reaction is as if the projectile neutron had merely "bounced off," or scattered from, the nucleus. The two categories of scattering reactions, elastic and inelastic scattering are described in the following paragraphs.
2.4.1 Elastic Scattering

In an elastic scattering reaction between a neutron and a target nucleus, there is no energy transferred into nuclear excitation. Momentum and kinetic energy of the "system" are conserved although there is usually some transfer of kinetic energy from the neutron to the target nucleus. The target nucleus gains the amount of kinetic energy that the neutron loses. In the elastic scattering reaction, the conservation of momentum and kinetic energy is represented by the equations below.

i. Conservation of momentum (mv)

\[(m_n v_{n,i}) + (m_T v_{T,i}) = (m_n v_{n,f}) + (m_T v_{T,f})\]  \hspace{1cm} (2.3)

ii. Conservation of kinetic energy \(\left(\frac{1}{2}mv^2\right)\)

\[\left(\frac{1}{2}m_n v_{n,i}^2\right) + \left(\frac{1}{2}m_T v_{T,i}^2\right) = \left(\frac{1}{2}m_n v_{n,f}^2\right) + \left(\frac{1}{2}m_T v_{T,f}^2\right)\]  \hspace{1cm} (2.4)

Where:

- \(m_n\) = mass of the neutron
- \(m_T\) = mass of the target nucleus
- \(v_{n,i}\) = initial neutron velocity
- \(v_{T,i}\) = initial target velocity
- \(v_{n,f}\) = final neutron velocity
- \(v_{T,f}\) = final target velocity

Elastic scattering of neutrons by nuclei can occur in two ways. The more unusual of the two interactions is the absorption of the neutron, forming a compound nucleus, followed by the re-emission of a neutron in such a way that the total kinetic energy is conserved and the nucleus returns to its ground state.
This is known as resonance elastic scattering and is very dependent upon the initial kinetic energy possessed by the neutron. Due to formation of the compound nucleus, it is also referred to as compound elastic scattering. The second, more usual method is termed potential elastic scattering and can be understood by visualizing the neutrons and nuclei to be much like billiard balls with impenetrable surfaces. Potential scattering takes place with incident neutrons that have energy of up to about 1 MeV. In potential scattering, the neutron does not actually touch the nucleus and a compound nucleus is not formed. Instead, the neutron is acted on and scattered by the short range nuclear forces when it approaches close enough to the nucleus [4].

2.4.2 Inelastic Scattering

In inelastic scattering the incident neutron is absorbed by the target nucleus, forming a Compound nucleus. The compound nucleus will then emit a neutron of lower kinetic energy which leaves the original nucleus in an excited state. The nucleus will usually by one or more gamma emissions emit this excess energy to reach its ground state.

For the nucleus that has reached its ground state, the sum of the kinetic energy of the exit neutron, the target nucleus, and the total gamma energy emitted is equal to the initial kinetic energy of the incident neutron [3].

2.5 Nuclear Fission

This causes the nucleus to behave like a liquid drop. Therefore, if energy is applied from the outside, oscillation modes are excited in the same way as for a liquid drop. In this way, a nucleus can fission into two pieces, which is the process known as nuclear fission. This type of nuclear fission is observed in heavy nuclei.
Among the naturally-occurring nuclides, only $^{235}$U can fission by thermal neutrons. If the energy of the neutrons is increased, $^{238}$U and $^{232}$Th can fission. Nuclear fission by a thermal neutron is called thermal fission and that by a fast neutron is called fast fission. Thermally fissionable material is called fissile material, and includes $^{233}$U, $^{239}$Pu, and $^{241}$Pu in addition to $^{235}$U. The nuclide $^{233}$U can be prepared from $^{232}$Th, and $^{239}$Pu, and $^{241}$Pu can be prepared from $^{238}$U by neutron absorption. Material that can produce fissile material is called fertile material. The nuclide $^{252}$Cf can fission without being irradiated by neutrons, a type of nuclear fission called spontaneous fission.

The only naturally-occurring elements that are useful as nuclear fuel are uranium and thorium. Uranium of natural composition is called natural uranium. Natural uranium contains 0.0054% $^{234}$U, 0.720% $^{235}$U, and 99.275% $^{238}$U. Natural thorium contains 100% $^{232}$Th. When nuclear fission takes place, the nucleus splits into two fission fragments of approximately equal mass. Fission fragments are usually called fission products. This corresponds to about 200 MeV for the total number of nucleons, which is the energy generated by nuclear fission.

If we assume that a nucleus splits into fission products of approximately equal mass by nuclear fission, the fission products would have too many neutrons compared with stable nuclei, Therefore neutrons are usually generated at the time of nuclear fission.

The number of generated neutrons increases as the energy of the colliding neutron increases or the mass number of the nucleus increases. When a thermal neutron is incident on $^{235}$U, about 2.5 neutrons are generated [17].
2.6 Principle of chain reactions

Chain reactions are an everyday concept. For example, fire is a chain reaction in which heat causes a chemical reaction (combustion) that produces heat, which causes combustion to continue, producing more heat, and so forth. As mentioned above, when physicists discovered that neutron-induced fission also emitted a few neutrons, they realised that chain reactions were a possibility:

\[
\text{Neutrons} \Rightarrow \text{Fissions} \Rightarrow \text{Neutrons} \Rightarrow \text{Fissions} \Rightarrow \text{Neutrons} \Rightarrow \text{etc.}
\]

Such a reaction would release a phenomenal amount of energy, which could be used either for peaceful purposes or to create a formidable weapon. To produce energy for peaceful purposes, the rate of reaction must be controlled, as it would be in a classic boiler. For a weapon, as in a bomb using chemical explosives, fast amplification of the reaction is required.
2.7 Chain Reactions in nuclear reactors

We could get around this problem if we had a substance that produced free neutrons as a result of its being bombarded with neutrons. In other words, we need a substance that produces the catalyst (bombarding neutrons) from a reaction that was caused by the catalyst. It turns out that we are in luck, in this regard, as there is one naturally occurring isotope that is abundant enough to run a power plant. When uranium-235 (U-235) is bombarded with low energy neutrons, its nucleus will fragment into several parts, with neutrons being amongst them. A typical reaction for U-235 (one of many possibilities, all of which produce neutrons) is

\[ \text{Neutron} + \text{uranium-235} \rightarrow \text{barium-144} + \text{krypton-89} + 3 \text{ neutrons} + 173.3 \text{ MeV} \]

The three neutrons that are released by this reaction are free to bombard three other uranium-235 nucleuses which would then decay into barium and krypton, on fragments with up to 9 more neutrons and about three times the amount of energy being released.

This chain reaction would look something like the picture below. There are two features about this chain reaction that require further discussion. The first of these is that it is only slowly moving neutrons that have a high probability interacting with the uranium-235 nuclei and causing a reaction. Therefore, a neutron moderator which slows the neutrons down, is needed in a nuclear power plant in order to keep the chain reaction occurring. The second feature has to do with the number of neutrons that will be present after several different decays. While there are many different possible decay reactions that could take place uranium-235 nucleus will average about 2.5 neutrons released in each one.
This means that each reaction will produce more neutrons than what were there initially, thus causing more reactions to take place at the next stage. If this is allowed to go on for some time, there will be so many neutrons around ready to react that too many decays will start taking place, which will release too much energy and cause the material to melt down. For this reason control rods, which are made from materials that readily absorb neutrons, need to be in the system in order to limit the number of reactions that can take place at any given time [17].

Fig (2.3): U-235 undergoing a chain reaction in the reactor
2.8 Neutron Flux

The neutron flux is a quantity used in nuclear reactor physics corresponding to the total length travelled by all neutrons per unit time and volume, or nearly equivalently number of neutrons travelling through unit area in unit time. The neutron fluence is defined as the neutron flux integrated over a certain time period [17].

The neutron flux value is calculated as the neutron density \( n \) multiplied by neutron velocity \( v \), where \( n \) is the number of neutrons per cubic centimeter (expressed as neutrons/cm\(^3\)) and \( v \) is the distance the neutrons travel in 1 second (expressed in centimeters per second, or cm/sec)[18]. Consequently, neutron flux \( (nv) \) is measured in neutrons/cm\(^2\)/sec. The neutron flux \( \phi \) is the number of neutrons \( (n) \) multiplied by their velocity \( (v) \) [6]:

\[
\phi = nv
\]  

(2.5)

2.9 Mechanism of Fast Neutron Moderation & Thermalization

Scattering events can be subdivided into elastic and inelastic scattering. In elastic scattering the total kinetic energy of the neutron and nucleus is unchanged by the interaction. During the interaction, a fraction of the neutron’s kinetic energy is transferred to the nucleus. For a neutron of kinetic energy \( E \) encountering a nucleus of atomic weight \( A \), the average energy loss is \( 2EA/(A + 1)^2 \). This expression shows that in order to reduce the speed of neutrons (that is, to moderate them) with the fewest number of elastic collisions target nuclei with small \( A \) should be used. By using hydrogen, with \( A= 1 \), the average energy loss has its largest value of \( E/2 \).
A neutron with 2 MeV of kinetic energy will (on the average) have 1 MeV left after one elastic collision with a hydrogen nucleus, 0.5 MeV after a second such collision, and so on. To achieve a kinetic energy of only 0.025 eV would take a total of about 27 such collisions. (A neutron of energy 0.025 eV is roughly in thermal equilibrium with its surrounding medium and is considered a “thermal neutron”.

From the relation \( E = KT \) where \( K \) is Boltzmann’s constant, an energy \( E \) of 0.025 eV corresponds to a temperature \( T \) of 20°C. In general, after \( n \) elastic collisions, the neutron’s energy is expected to change from \( E_0 \) to \( E_n = E_0[A^2 + 1/(A + 1)^2]^n \). To reach \( E_n \) from \( E_0 \) thus requires \( n = \log [A^2 + 1/(A + 1)^2] \) collisions, on the average.

Table (2.1) gives examples of the number of collisions required to “thermalize” a 2MeV neutron in some materials. Inelastic scattering is similar to elastic scattering except that the nucleus undergoes an internal rearrangement into an excited state from which it eventually releases radiation.

The total kinetic energy of the outgoing neutron and nucleus is less than the kinetic energy of the incoming neutron; part of the original kinetic energy is used to place the nucleus into the excited state. It is no longer easy to write an expression for the average energy loss because it depends on the energy levels within the nucleus. But the net effect on the neutron is again to reduce its speed and change its direction. If all the excited states of the nucleus are too high in energy to be reached with the energy available from the incoming neutron, inelastic scattering is impossible. In particular, the hydrogen nucleus does not have excited states, so only elastic scattering can occur in that case [3].
In general, scattering moderates or reduces the energy of neutrons and provides the basis for some neutron detectors (for example, proton recoil detectors) [3].

Table (2.1): Average number of collisions required to reduce a neutron’s energy from 2MeV to 0.025 eV by elastic scattering [4]

<table>
<thead>
<tr>
<th>Element</th>
<th>Atomic Weight</th>
<th>Number of collisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deuterium</td>
<td>2</td>
<td>31</td>
</tr>
<tr>
<td>Helium</td>
<td>4</td>
<td>48</td>
</tr>
<tr>
<td>Beryllium</td>
<td>9</td>
<td>92</td>
</tr>
<tr>
<td>Carbon</td>
<td>12</td>
<td>119</td>
</tr>
<tr>
<td>Uranium</td>
<td>238</td>
<td>2175</td>
</tr>
</tbody>
</table>

2.10 Logarithmic energy decrement ($\xi$)

A convenient measure of energy loss per collision is the logarithmic energy decrement. The average logarithmic energy decrement is the average decrease per collision in the logarithm of the neutron energy. This quantity is represented by the symbol (Greek letter xi).

$$\xi = \ln E_i - \ln E_f$$ (2.6)
\[ \xi = \ln \left( \frac{E_i}{E_f} \right) \]  

(2.7)

Where:

\( \xi \) = average logarithmic energy decrement

\( E_i \) = average initial neutron energy

\( E_f \) = average final neutron energy

The symbol is commonly called the average logarithmic energy decrement because of the fact that a neutron loses, on the average, a fixed fraction of its energy per scattering collision. Since the fraction of energy retained by a neutron in a single elastic collision is a constant for a given material, \( \xi \) is also a constant. Because it is a constant for each type of material and does not depend upon the initial neutron energy, \( \xi \) is a convenient quantity for assessing the moderating ability of a material [10]. The values for the lighter nuclei are tabulated in a variety of sources. The following commonly used approximation may be used when a tabulated value is not available.

\[ \xi = \frac{2}{A+\frac{2}{3}} \]  

(2.8)

This approximation is relatively accurate for mass numbers (A) greater than 10, but for some low values of A it may be in error by over three percent. Since \( \xi \) represents the average logarithmic energy loss per collision, the total number of collisions necessary for a neutron to lose a given amount of energy may be determined by dividing into the difference of the natural logarithms of the energy range in question.
The number of collisions (N) to travel from any energy, $E_{\text{high}}$, to any lower energy, $E_{\text{low}}$, can be calculated as shown below[10]:

$$N = \frac{\ln E_{\text{high}} - \ln E_{\text{low}}}{\xi}$$  \hspace{1cm} (2.9)$$

$$N = \ln \left( \frac{E_{\text{high}}}{E_{\text{low}}} \right)$$  \hspace{1cm} (2.10)$$

### 2.11 Moderating Ratio & Moderating Power

A standard basis for comparing moderating abilities of different materials is the Moderating power. If one material has a larger moderating power than another, less of that material is needed to achieve the same degree of moderation. Two factors are important: (1) the probability of a scattering interaction and (2) the average change in kinetic energy of the neutron after such an interaction. To be an effective moderator, both the probability of an interaction and the average energy loss in one scatter should be high. The moderating power is defined as $\xi_s$ where $s$ is the macroscopic scattering cross section and $\xi$ is the average logarithmic energy decrement in a scatter.

This decrement is $\ln(E_{\text{before}}) - \ln(E_{\text{after}})$. When elastic collisions in an element with atomic weight $A$ dominate the scattering process, the decrement becomes

$$\xi = 1 - \frac{(A-1)^2}{2A} \ln \left( \frac{A+1}{A-1} \right)$$  \hspace{1cm} (2.11)$$
For $A>2$, $\xi$ can be approximated by $2/(A + 0.67)$. The moderating power of a compound is given by

$$\xi_s = \frac{\rho N_a}{M} (n_1 \sigma_1 \xi_1 + n_2 \sigma_2 \xi_2 + \ldots)$$  \hspace{1cm} (2.12)

Where $\rho$ is the density of the compound, $M$ is its molecular weight, $N_a$ is Avogadro’s number, $n_i$ is the number of atoms of element $i$ in one molecule, $\sigma_i$ is the microscopic scattering cross section for element $i$, and $\xi_i$ is the logarithmic energy decrement for element $i$.

A material with a large moderating power might never the less be useless as a practical moderator if it has a large absorption cross section. Such a moderator would effectively reduce the speeds of those neutrons that are not absorbed, but the fraction of neutrons that survive may be too small to be used in a practical manner. A more comprehensive measure of moderating materials is the moderating ratio $\xi_s/\rho$. A large moderating ratio is desirable; it implies not only a good moderator but also a poor, absorber. For a compound, the moderating ratio is given by Equation (2.4) with each $\sigma_i$ replaced by $\sigma_s/\sigma_a$ for element $i$.

Table (2.2) gives the moderating powers and ratios for some common moderator materials for neutrons in the $1\text{-eV}$ to $100\text{-keV}$ energy range. Ordinary water has a higher moderating power than heavy water because the atomic weight of hydrogen is half that of deuterium.

But the hydrogen nucleus (a proton) can absorb a neutron and create deuterium much more readily than a deuterium nucleus can absorb a neutron and create tritium. This difference in absorption cross sections gives heavy water a much more favorable moderating ratio.
However, because of its availability and low cost, ordinary water is often preferred. The solid materials given in the table have a higher moderating ratio than ordinary water and can have fabrication advantages. Polyethylene is commonly selected as a moderator because of its high moderating power and moderating ratio [4].

<table>
<thead>
<tr>
<th>Moderator</th>
<th>Moderating Power (1eV to 100 KeV)</th>
<th>Moderating Ratio (Approximate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.28</td>
<td>58</td>
</tr>
<tr>
<td>Heavy Water</td>
<td>0.18</td>
<td>21000</td>
</tr>
<tr>
<td>Helium at STP</td>
<td>0.00001</td>
<td>45</td>
</tr>
<tr>
<td>Beryllium</td>
<td>0.16</td>
<td>130</td>
</tr>
<tr>
<td>Graphite</td>
<td>0.064</td>
<td>200</td>
</tr>
<tr>
<td>Polyethylene (CH₂)</td>
<td>3.26</td>
<td>122</td>
</tr>
</tbody>
</table>

Table (2.3): Comparison of the main moderator nuclei (the cross-section is taken for epithermal neutrons [a few eV] and the absorption cross-section for thermal neutrons [0.0253 eV]) [9].

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Mass</th>
<th>Average Lethargy gain</th>
<th>Scattering cross-section</th>
<th>Absorption cross-section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>1.00</td>
<td>1.0000</td>
<td>20.4</td>
<td>0.332</td>
</tr>
<tr>
<td>Deuterium</td>
<td>2.00</td>
<td>0.7261</td>
<td>3.40</td>
<td>0.00051</td>
</tr>
<tr>
<td>Beryllium</td>
<td>8.93</td>
<td>0.2080</td>
<td>6.00</td>
<td>0.0076</td>
</tr>
<tr>
<td>Carbon</td>
<td>12.01</td>
<td>0.1589</td>
<td>4.74</td>
<td>0.00337</td>
</tr>
<tr>
<td>Oxygen</td>
<td>15.86</td>
<td>0.1209</td>
<td>3.89</td>
<td>0.000191</td>
</tr>
</tbody>
</table>
Table (2.4): Comparison of the main moderator materials [9]

<table>
<thead>
<tr>
<th>Material</th>
<th>Density</th>
<th>Concentration</th>
<th>Moderating power</th>
<th>Relative (material/water)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary</td>
<td>998</td>
<td>0.03337</td>
<td>137.72</td>
<td>1</td>
</tr>
<tr>
<td>Heavy water</td>
<td>1105</td>
<td>0.0332</td>
<td>17.95</td>
<td>0.130</td>
</tr>
<tr>
<td>Beryllium</td>
<td>1850</td>
<td>0.124</td>
<td>15.48</td>
<td>0.112</td>
</tr>
<tr>
<td>Beryllia</td>
<td>3010</td>
<td>0.0725</td>
<td>12.46</td>
<td>0.090</td>
</tr>
<tr>
<td>Graphite</td>
<td>1600</td>
<td>0.0802</td>
<td>6.04</td>
<td>0.044</td>
</tr>
</tbody>
</table>

2.12 Control Rods

Consider now the stability of the chain reaction. This is where the control rods play their part. They are usually made of cadmium, which has a very high absorption cross-section for neutrons. By mechanically manipulating the control rods, i.e. by retracting or inserting them, the number of neutrons available to induce fission can be regulated. This mechanism is the key to maintaining a constant $k$ value of unity and therefore a constant power output. However, safe working of the reactor is not possible with prompt neutrons alone [14].
Chapter Three

Monte Carlo Modeling
3.1 Monte Carlo code

The MCNP5 Code, developed and maintained by Los Alamos National Laboratory, is the internationally recognized code for analyzing the transport of neutrons and gamma rays (hence NP for neutral particles) by the Monte Carlo method (hence MC). The code deals with transport of neutrons, gamma rays, and coupled transport, i.e., transport of secondary gamma rays resulting from neutron interactions. The MCNP code can also treat the transport of electrons, both primary source electrons and secondary electrons created in gamma-ray interactions [11].

3.1.1 Structure of the MCNP5 Input File

An input file has the structure. Input lines have a maximum of 80 columns and command mnemonics begin in the first 5 columns. Free field format (one or more spaces separating items on a line) is used and alphabetic characters can be upper, lower, or mixed case. A continuation line starts with 5 blank columns or a blank followed by the end of the card to be continued, [11].

3.1.2 Geometry Specifications

MCNP treats problem geometry primarily in terms of regions or volumes bounded by first and second degree surfaces. Cells are defined by intersections unions and complements of the regions and contained user defined materials, [11].
3.2 The Proposed Source

In this work we utilized Californium source $^{252}\text{Cf}$ as the neutron source. The spectrum of the simulated $^{252}\text{Cf}$-neutron source and the basic information about it are shown in figure 3.1 and table (3.1).

Table (3.1): Basic information for $^{252}\text{Cf}$ Source [14].

<table>
<thead>
<tr>
<th>Source</th>
<th>Production</th>
<th>Half-life</th>
<th>Type of Emission</th>
<th>Neutron emission rate ($s^{-1}/\mu g$)</th>
<th>Specific Activity ($\text{mCi}/\mu g$)</th>
<th>Average neutron energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{252}\text{Cf}$</td>
<td>Nuclear reactors</td>
<td>2.645 years</td>
<td>Primary: Neutron, Secondary: Gamma</td>
<td>$2.314 \times 10^6$</td>
<td>0.536</td>
<td>2.1</td>
</tr>
</tbody>
</table>
The peak of the curve in the spectrum of $^{252}$Cf without moderator is $2.5\text{MeV}$ and neutron flux is $0.115\text{n.m}^{-2}\text{s}^{-1}$.

Fig (3.1): Simulated Neutron Energy Spectrum of $^{252}$Cf
3.3 Model of the Control System

The geometry of the reactor-like environment in this work is composed of four items; the first is a moderating material in the form of a cube of dimensions 90cm x 90cm x 90cm being contained with the second item which is a carbon reflector of dimensions 100cm x 100cm x 100cm, suggesting a reflector with thickness 10cm. The third item is a neutron detector in a form of ring surrounding the interior circumference of the moderator having radius of 80cm. The fourth item is a material of high absorption cross section for thermal neutrons, simulated as control rods with 80 cm length and 1 cm radius.

Fig (3.2) Show the components and geometry of the proposed Control system without control rods as featured in the computer code. In this figure, the reflector appears with purple color which represents carbon ($^{14}$C) as reflector for fast neutrons. The moderator appears with blue color. Fig (3.3) shows the components and geometry of the proposed Control system with control rods. In this figure the control rods appear with yellow colors.
Fig (3.2): Model of the Control system without control rods as modeled in MCNP simulations
Fig (3.3): Model of the proposed control system with control rods as modeled in MCNP simulations
3.4 Materials Used

i. Properties of moderator & absorbers

Table (3.2) and Table (3.3) contain the moderating materials and absorbers used in our current work, respectively.

**Table [3.2]: Properties of materials moderator**

<table>
<thead>
<tr>
<th>Material/Density g/cm³</th>
<th>Mass fractions</th>
<th>O</th>
<th>H</th>
<th>C</th>
<th>B</th>
<th>D</th>
<th>Be</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy water (D₂O) 1.11</td>
<td></td>
<td>0.888101</td>
<td></td>
<td></td>
<td></td>
<td>0.111898</td>
<td></td>
</tr>
<tr>
<td>Light water (H₂O) 0.998</td>
<td></td>
<td>0.889</td>
<td>0.111</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table [3.3]: Properties of materials absorber**

<table>
<thead>
<tr>
<th>B-10</th>
<th>B-11</th>
<th>Ag-105</th>
<th>Ag-107</th>
<th>Cd-113</th>
</tr>
</thead>
<tbody>
<tr>
<td>3835</td>
<td>0.006</td>
<td>38</td>
<td>91</td>
<td>20600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Natural Abundance (%)</th>
<th>20</th>
<th>80</th>
<th>52</th>
<th>48</th>
<th>12</th>
</tr>
</thead>
</table>
Chapter Four

Results and Discussion
CHAPTER FOUR
RESULTS AND DISCUSSION

4.1 Results for (H₂O) & (D₂O) as moderators without Control Rods

In this scenario, light water (H₂O) & Heavy water (D₂O) were utilized as moderators for fast neutrons and their performance in thermalizing fast neutrons of ²⁵²Cf-source was examined. The results are plotted in figure (4.1)

Fig (4.1): Neutron flux as a function of neutron energy in the presence of light water (H₂O) & Heavy water (D₂O)
As shown in figure (4.1), When the simulated neutrons of the $^{252}$Cf-source (supposed to produce $10^5$ neutrons.s$^{-1}$ with average energy 2.5 MeV) interact with Light water (H$_2$O) or heavy water (D$_2$O), for both moderators, the curve peaks $1.88 \times 10^{-7}$ Mev. The different between two results lies in the neutron flux; in the case of light water (H$_2$O) its value is $5.0953 \times 10^{-6}$ n.m$^{-2}.s^{-1}$ and in the case of heavy water (D$_2$O) its value is $5.58 \times 10^{-5}$ n.m$^{-2}.s^{-1}$. Thus the neutron flux for (D$_2$O) is higher than that of (H$_2$O) by about 90%. That’s why we choose heavy water as our optimal moderator.

4.2 Results for (D$_2$O) as moderator with cadmium (Cd) as control Rods

The performance of cadmium (Cd) as control material in the presence of the optimal moderator (Heavy water (D$_2$O)) was examined; the results are plotted in figure (4.2)
Fig (4.2): Neutron flux as a function of neutron energy, D$_2$O+Cd

The neutron flux peaks in the thermal region ($1.88 \times 10^{-7}$ Mev), with a value of $5.12 \times 10^{-4} \text{ n.m}^{-2} \text{s}^{-1}$.

4.3 Results for (D$_2$O) as moderator with silver (Ag) as control Rods

In this scenario, utilize Heavy water (D2O) as moderators for fast neutron, and silver (Ag) as control material. The results are shown in figure (4.3).
Fig (4.3): Neutron flux as a function of neutron energy, D$_2$O+Ag

The neutron flux peaks in the thermal region (1.88×10$^{-7}$Mev), with a value of 3.9×10$^{-4}$ n.m$^{-2}$.s$^{-1}$.

4.4 Results for (D$_2$O) as moderator with Boron carbide (BC$_4$) as control Rods

In this scenario, utilize Heavy water (D$_2$O) as moderators for fast neutron, and Boron carbide (BC$_4$) as control material, the results are plotted in figure (4.4).
Fig (4.4): Neutron flux as a function of neutron energy, $\text{D}_2\text{O}+\text{BC}_4$

The neutron energy peaks in the still thermal region $1.88\times10^{-7}\text{Mev}$, with a value of $2.9\times10^{-4}\text{ n.m}^{-2}\text{.s}^{-1}$. 
4.5 Comparison of Results

As shown in figure (4.6) since the moderating material for all configurations is D$_2$O, The neutron flux for all configurations peaks at the same energy $1.88 \times 10^{-7}$ Mev. The difference in the neutron flux; the neutron flux for (D$_2$O+BC$_4$) configuration is lower than that of (D$_2$O+Cd) and (D$_2$O+Ag) configurations by about 43% and 25% respectively. These results suggest that BC$_4$ has effectively reduced the thermal neutron flux and consequently, (D$_2$O+BC$_4$) configuration is the optimal model for the proposed control system.

Fig (4.5): comparison between control materials
Thus \((D_2O+BC_4)\) has been chosen as the optimal configuration since \(D_2O\) reduces the energy of fast neutrons to thermal range and provide sufficient thermal neutron flux for the chain reaction, while \(BC_4\) works as an effective controller due to its ability to reduce the thermal neutron flux.

### 4.6 Conclusions

In this work, light water (H\(_2\)O) and heavy water (D\(_2\)O) are employed as moderating materials an. Carbon \(^{14}\)C was employed as reflector to prevent the leakage for neutrons outside the simulated nuclear core. Cadmium (Cd), silver (Ag), & Boron carbide (BC\(_4\)) was employed as absorbers. The results approved that heavy water (D\(_2\)O) is the optimal moderator, where (D\(_2\)O+BC\(_4\)) configuration is the optimal model for the proposed control system.

### 4.7 Recommendations

The research involved in this study, present MCNP simulations for the current proposed control system for nuclear thermal reactors and reviews the interaction mechanisms of neutrons with several moderators and absorbers. Further experimental verifications are required and other moderating materials and absorbers should be tested.
References


Appendix

i. Input file used in the current Monte Carlo Simulations
(Heavy Water D\textsubscript{2}O & Boron Carbide BC\textsubscript{4})

c CELL CARDS
1  1  -2.23  1  -2  -3  4  -5  6 (-7 :8 :9 :-10 :11 :-12 )
2  2  -1.11  7  -8  -9  10 -11 12 (16 :-20 :21 )(17 :-20 :21 )
(18 :-20 :21 )
3  3  -2.52  -16 20 -21
4  3  -2.52  -17 20 -21
5  3  -2.52  -18 20 -21
6  3  -2.52  -19 20 -21
7  4  -0.0012 (-1 :2 :3 :-4 :5 :-6 )-15
8  0  15

c CERFACE CARDS
1   pz  -50
2   pz   50
3   py  100
4   py   0
5   px   50
6   px  -50
7   pz  -40
8   pz   40
9   py   90
10  py  10
11  px  40
12  px -40
15  so  200
16  c/y  3  0  1
17  c/y -3  0  1
18  c/y  10  0  1
19  c/y -10  0  1
20  py  12
21  py  88

c MATERIAL SPECIFICATION
m1  6012. -1 $MAT
m2  1002. -0.111898 $MAT
8016.  -0.888101
m3  5010.  -0.769  $MAT
     6012.  -0.231
m4  7014.  -0.78  $MAT
     8016.  -0.22

c DATA CARDS

c CELL IMPORTANCE
imp:n  1 6r
      0 $ 1, 8

c SOURCE SPECIFICATION
sdef  pos= 0 50 0  erg=d1
sil  3.16000E-02  3.98000E-02  5.01000E-02  6.31000E-02
     7.94000e-02  1.00000E-01  1.26000E-01  1.58000E-01
     2.00000e+00  1.58000E+00  2.00000E+00  2.51000E+00
     3.16000e+00  3.98000E+00  5.01000E+00  6.31000E+00
     7.94000e+00  1.00000E+01  1.26000E+01  1.58000E+01
     2.00000e+01
spl  0.00E+00  3.58E-05  1.97E-04  3.58E-04
     1.02E-03  1.86E-03  2.91E-03  4.59E-03
     6.42E-03  1.13E-02  1.81E-02  2.37E-02
     3.08E-02  4.15E-02  5.38E-02  6.53E-02
     8.07E-02  9.57E-02  1.07E-01  1.12E-01
     1.06E-01  8.99E-02  6.87E-02  4.43E-02
     2.22E-02  8.30E-03  2.68E-03  4.47E-04
     7.09E-05

c TALLIES
f5y:n  50 35  0.5
e5  2.50000E-08  1.88000E-07  2.50000E-07  5.00000E-07
     1.00000E-06  2.15000E-06  4.65000E-06  1.00000E-05
     2.15000E-05  4.65000E-05  1.00000E-04  2.15000E-04
     4.65000E-04  1.00000E-03  2.15000E-03  4.65000E-03
     1.00000E-02  1.26000E-02  1.58000E-02  2.00000E-02
     2.51000E-02  3.16000E-02  3.98000E-02  5.01000E-02
     6.31000E-02  7.94000E-02  1.00000E-01  1.26000E-01
     1.58000E-01  2.00000E-01  2.51000E-01  3.16000E-01
     3.98000E-01  5.01000E-01  6.31000E-01  7.94000E-01
     1.00000E+00  1.26000E+00  1.58000E+00  2.00000E+00
     2.51000E+00  3.16000E+00  3.98000E+00  5.01000E+00
     6.31000E+00  7.94000E+00  1.00000E+01  1.26000E+01
     1.58000E+01  2.00000E+01
c
  c PROBLEM CUT-OFF
nps   10000
c
  c PERIPHERAL CARDS
print