1.1 Overview

The range of applications for electro-hydraulic systems is diverse, and includes manufacturing systems, materials test machines, active suspension systems, mining machinery, flight simulation, paper machines, ships and electromagnetic marine engineering, injection molding machines, robotics, and steel and aluminum mill equipment. Hydraulic systems are also common in aircraft, where their high power-to-weight ratio [1] and precise control makes them an ideal choice for actuation of flight surfaces.

Nowadays electro-hydraulic actuators are very important tools for industrial processes. This is mainly due to their fast response, high force output, large stroke, and high accuracy. These features are not easily matched for any other commercial technology used today in actuators construction. However, the control of electro-hydraulic systems can be a difficult problem since their dynamics are highly nonlinear. Therefore, the investigation of the position or force control for electro-hydraulic actuators should be of great interest from both academic and industrial perspectives. An important parameter which characterizes the performance of an actuator is its force output, which is defined as the amount of force that a given actuator can exert on an external body along or around its output axis. Hydraulic systems in general can generate a large output force compared to electric or pneumatic systems due to the high pressure of operation and the relative lack of compressibility of the working fluid.

Another parameter characterizes the performance of an actuator is its stroke, which is defined as the maximum amount of extension of a linear actuator or the maximum amount of rotation
of a rotary actuator. It is more commonly used to describe linear actuators than rotary actuators because many of the latter can rotate indefinitely and therefore do not have a fixed stroke.

Finally, the most important performance characteristic of a high-accuracy actuator is its accuracy. Accuracy is a general term, and one must ensure that the circumstance under which it is measured is included in the specification. For example, an actuator may claim certain accuracy without mentioning that the measurement is only valid when the actuator is unloaded. Additionally, accuracy can be specified in both a static and dynamic sense. For example, static accuracy could mean the steady-state error when the actuator is required to move a certain distance, while dynamic accuracy could refer to the accuracy of an actuator when it is tracking a varying input signal.

One important property of the hydrostatic systems is the use of symmetric actuators. Here, assuming the leakages are compensated, the input flow rate of the variable displacement pump or variable speed pump will be equal to the output flow rate of the actuator making the control very simple. However if an asymmetric single rod cylinder is used as the hydraulic actuator, then the flow entering the actuator will not be equal to the flow exiting from the actuator. To overcome this problem, a novel symmetric single rod actuator design is presented by Goldenberg and Habibi [1]. However, manufacturing of this new design necessitate more precision than the simple single rod cylinder and introduce more manufacturing cost. To compensate the asymmetric flow rate of a single rod hydraulic actuator, hydraulic transformers are utilized. A hydraulic transformer converts an input flow at a certain given pressure to an output flow at any other pressure level. Here, the product of pressure and flow at the input is equal to the product of pressure of flow at the output. It can be compared to an electric transformer where the product of voltage and current in principle remains constant [5]. In 1988,
Berbuer introduced a hydraulic transformer for the volume flow compensation of the single rod cylinder. The ratio of the transformer is designed according to the single rod cylinder area ratio [6].

Several examples of applications which require a combination of high force output, large stroke and high accuracy can be given. The first is positioning of runner blades in Kaplan generation units, regarding water head, and guide vane opening to maintain constant shaft speed. This is critical if the unit is taking care of power network frequency. The response and accuracy characteristics here, affects the efficiency of the generation unit.

Another example of application that needs extreme accuracy and high force output is micro-machining and high-precision grinding [2], [3]. In this application, the actuator is used to position a cutting or grinding tool against a rotating blank of material. The accuracy of the final part will depend directly on the characteristics of the actuator, causing this to be one of the most demanding applications of any actuator. Those examples explain the need for an actuation system which can position large inertial loads with high accuracy and is capable of significant actuator displacements. This needs and difficulties in implementation was the motivator for the project presented in this thesis.
1.2 Problem Statement

In order to investigate the current performance level of the hydro turbine governor compared to installed performance level directly, turbine should be in outage for relatively long time; this will impose a considerable loss of electric power. Moreover, extra measuring devices are needed, which will increase the cost of performance tests.

After major rehabilitation process of turbine parts, and Rosaries Dam heightening project, we may need to re-adjust the turbine speed governor, initially it is not practically to do this on-line.

1.3 Proposed Solution

A non-linear model based on computerized modeling that exhibits realistic behavior of the plant governor actuator will be useful for dynamic analysis of the turbine speed governor. This model can also be used for explanation purposes for new engineers and university students.

1.4 Methodology

MATLAB/Simulink software environment, is one of the favorite and trusted software for modeling and simulation of control systems, there for I relied mainly on it for the design and simulation of the electro hydraulic actuator (EHA).

1.5 Objectives

The main objective of this thesis is the derivation of a reliable model for Rosaries Hydro Power Station electro hydraulic actuator, which can be used for the following tasks;

- Turbines stability studies.
- Analyzing the impacts of dam heightening and turbine major rehabilitation activities on unit performance.
- Demonstrative tool for young engineers and universities students how annually visits the power station.

1.6 Thesis outlines

This thesis is arranged in a manner that describes the progression of the research project process. In chapter 2, background review of Hydraulic Actuators is presented, including basics of hydraulic systems. Chapter 3 presents mathematical modeling of the targeted RHPS actuator, steady state and dynamic characteristics of it. Chapter 4 presents the controllers modeling procedures. Chapter 5 describes simulation results comparisons with the real actuator, including step response and open-loop close-loop test results. Chapter 6 presents discussion, conclusions of this thesis, and recommendations for future works.


Chapter 2

Background

2.1 Introduction

The basic principle of hydrostatic machines and systems is based on Pascal’s law formulated in the 17th century. It states: “…Any change of pressure at any point on an incompressible fluid at rest, which does not disturb the equilibrium of the fluid, is transmitted to all other points of the fluid without any change. When forces of pressure balance the gravitational forces, then the pressure at every point of the fluid is the same…” It took almost 150 years till Pascal’s ideal found practical application during the industrial revolution [7]. Over the years, the integration of electronics into hydraulics has lead to further modernization of hydrostatic systems. A hydrostatic system includes several components including pumps, valves, pipelines etc.

The range of applications for electro-hydraulic servo systems is diverse, and includes Manufacturing systems, materials test machines, active suspension systems, mining machinery, fatigue testing, flight Simulation, paper machines, ships and electromagnetic marine engineering, injection molding machines, robotics, and steel and aluminum mill equipment. Hydraulic systems are also common in aircraft, where their high power-to-weight ratio and precise control makes them an ideal choice for actuation of flight surfaces.

Apart from the ability to deliver higher forces at fast speeds, servo-hydraulic systems offer several other benefits over their electrical counterparts. For example, hydraulic systems are mechanically “stiffer”, resulting in higher machine frame resonant frequencies for a given power level, higher loop gain
and improved dynamic performance. They also have the important benefit of being self-cooled since the driving fluid effectively acts as a cooling medium carrying heat away from the actuator and flow control components. Unfortunately hydraulic systems also exhibit several inherent non-linear effects, which can complicate the control problem.

Although electrical motors are sometimes used in many of those applications, motion control systems requiring either very high force or wide bandwidth are often addressed more efficiently with electro-hydraulic rather than electromagnetic means. In general, applications with bandwidths of greater than about 20 Hz or control power greater than about 15 kW may be regarded as suitable for servo-hydraulic techniques [8]. The bandwidth of the overall system is highly dependent upon the bandwidth of the electro hydraulic valve. A valve is called single stage if the valve actuator directly moves the main spool, two stage if the valve actuator motion is amplified by an intermediate pilot stage (i.e. double nozzle flapper servo valve, jet pipe servo valve, EH pilot actuated proportional valve) [9].

2.2 Valve Controlled Actuator circuit

A conventional hydraulic control system represented in Figure 2.1 consists of the following components:

- Hydraulic power system (Power source and pump).
- Relief valve
- Fluid reservoir
- Flow control valve
- Actuator
In the circuit illustrated in Figure 2.1, generally an AC electric motor is used as the power source. The motor drives a positive displacement pump. It is a common practice to use fixed displacement pumps since they are cheaper than other types of pumps. The fixed displacement pump is driven in one direction with constant speed; it sucks oil from the oil reservoir and delivers a constant flow rate through the hydraulic cylinder. The direction of motion of the hydraulic cylinder and its velocity are controlled by a flow control valve, which can be a proportional or servo valve. This valve regulates the flow by changing its orifice area. Assuming that the pressure drop across the valve is kept constant, there is a linear relationship between the flow rate and the orifice area. To retard or decelerate the hydraulic cylinder, the orifice area decreases, but this time as the valve resistance increases the pump exit pressure increases.
To keep the pressure below a maximum allowable operating pressure, a pressure relief valve is used at the pump outlet. This valve is normally closed, however, when the supply pressure reaches the set pressure of the relief valve, it opens and the excess flow returns to the oil tank through the relief valve. By this way, as long as an excess flow rate is delivered to the system, the relief valve will be always open limiting the pump exit pressure so that it does not affect by the changing valve orifices areas.

The circuit in Figure 2.1 is called as the "constant pressure (CP) valve controlled hydraulic system". The other type of the valve controlled hydraulic systems is the constant flow (CQ) systems. In constant pressure systems, the supply pressure to the control valve is kept constant whereas, in constant flow systems the rate of flow from the source through the control valve is kept constant. Therefore the supply pressure of the valve at any instant depends upon the conditions of operation at any time in CQ systems. The CP systems are the most popular one in hydraulic applications. Because the valve characteristics of CQ systems are highly non-linear compared with the CP systems, also with CQ systems it is not suitable to drive multi actuators from the same source [10].

2.3 Pressure relief Valve

A pressure relief valve fig 2.2 limits the maximum pressure in a system. The pressure relief valve consists of main valve housing (1) with main poppet assembly (3), and pilot valve (2) with a manually set pressure adjustment mechanism. Pressure relief valve Pressure at port A acts on the lower surface area of main poppet (3).
Simultaneously (internally piloted model), pressure from port “A” acts via control passages (10), (6), (7) and orifices (4), (5) on the spring loaded side of main poppet (3) plus pilot poppet (8). In the externally piloted model, pilot pressure may originate from port “X” (15) via passages (6), (7) and orifices (4) and (5). As pressure exceeds force on spring (9), pilot poppet (8) opens. This allows fluid in main poppet spring chamber and pilot passages, to drain via spring chamber (12) and passage (13) to port “B” (internally drained model) or via port “Y” (14) to tank (externally drained model). The pilot flow creates a pressure drop across orifices (4 and 5), allowing main poppet (3) to open. Fluid flows from port A to B and the valve modulates to maintain a pressure set value [2].
2.4 Electro Hydraulic Flow Control Valve

The electro-hydraulic flow control valve acts as a high gain electrical to hydraulic transducer, the input to which is an electrical voltage or current, and the output a variable flow of oil. The valve consists of a spool with lands machined into it figure 2.3, moving within a cylindrical sleeve. The lands are aligned with apertures cut in the sleeve such that movement of the spool progressively changes the exposed aperture size and alters differential oil flow between two control ports.

![Diagram of Three Land, Four-way Flow Control Valve Spool](image)

Figure 2.3: Diagram of Three Land, Four-way Flow Control Valve Spool
The ports are labeled P (pressure), T (tank), an, A and B (load control ports). The spool is shown displaced a small distance (Xv) as a result of a command force applied to one end, and arrows at each port indicate the direction of fluid flow which results. With no command force applied (Fv=0), the spool is centralized and all ports are closed off by the lands resulting in no load flow.

In the context of hydraulic servo-systems, flow control valves fall broadly into two main categories: proportional valves and servo-valves. Proportional valves use direct actuation of the main spool from an electrical force/torque motor, whereas servo-valves use at least one intermediate hydraulic amplifier stage between the electrical force/torque motor and the main spool. In this thesis we focus on the force motor actuated flow control valve.

A major advantage of proportional valves is that they are largely unaffected by changes in supply pressure and oil viscosity. However, the relatively large armature mass and large time constant associated with the coil means that these valves generally have poorer dynamic performance compared with servo-valves of equivalent flow characteristics. In recent years, “servo-proportional” valves have begun to appear with shorter spool displacements and lighter spools, giving dynamic performance which approaches that of true servo-valves but at a much lower cost.

The basic servo-valve produces a control flow proportional to input current for a constant load. While the dynamic performance of a servo-valve is influenced somewhat by operating conditions (supply pressure, input signal level, fluid and ambient temperature and so on) a major advantage is that load dynamics do not affect stability, unlike single stage proportional valves.
Servo valves usually have superior dynamic response, although their close internal machining tolerances make them relatively expensive and susceptible to contamination of the hydraulic fluid.

2.5 Hydraulic Supply Pressure

All hydraulic systems require a supply of pressurized fluid, usually a form of mineral oil. The choice of system oil pressure depends on various factors. Low pressure means less leakage, but physically larger components are required to develop a given force. High pressure systems suffer from more leakage, but have better dynamic performance and are both smaller and lighter.

Oil is drawn from a reservoir (tank) into a gear, rotary vane or piston pump, driven at constant speed by an electric motor. The oil is driven at constant flow rate into an adjustable pressure relief valve, which regulates system pressure by allowing excess oil to return to the reservoir once a pre-defined pressure threshold has been reached. Pressurized hydraulic oil is carried to the servo-valve through a system of rigid or flexible piping, possibly fitted with electrically operated shut-off valves to control hydraulic start-up and shut-down sequences. Oil is returned from the valve to the tank through a low pressure return pipe, which is often fitted with an in-line heat exchanger for temperature regulation of the oil.

One of the first steps in design of a hydraulic control system is to select supply pressure. Many considerations favor a large supply pressure. Power is the product of pressure and flow. As supply pressure is increased, less flow is required to provide a given power. Smaller Pump, lines, valves, oil supply etc are then possible. Faster response is often possible because of small oil volume and higher bulk modulus.
2.5.1 Screw Type Positive Displacement Pump

The two-screw, low-pitch screw pump consists of two screws that mesh with close clearances, mounted on two parallel shafts. One screw has a right-handed thread, and the other screw has a left-handed thread. One shaft is the driving shaft and drives the other shaft through a set of herringbone timing gears. The gears serve to maintain clearances between the screws as they turn and to promote quiet operation. The screws rotate in closely fitting duplex cylinders that have overlapping bores. All clearances are small, but there is no actual contact between the two screws or between the screws and the cylinder walls.

![Figure 2.4 Tow screws, low pitch pump](image)

The complete assembly and the usual flow path are shown in Figure 2.4. Liquid is trapped at the outer end of each pair of screws. As the first space between the screw threads rotates away from the opposite screw, a one-turn, spiral-shaped quantity of
liquid is enclosed when the end of the screw again meshes with the opposite screw. As the screw continues to rotate, the entrapped spiral turns of liquid slide along the cylinder toward the center discharge space while the next slug is being entrapped. Each screw functions similarly, and each pair of screws discharges an equal quantity of liquid in opposed streams toward the center, thus eliminating hydraulic thrust. The removal of liquid from the suction end by the screws produces a reduction in pressure, which draws liquid through the suction line.

### 2.5.2 Positive Displacement Pump Protection

Positive displacement pumps are normally fitted with relief valves on the upstream side of their discharge valves to protect the pump and its discharge piping from over pressurization. Positive displacement pumps will discharge at the pressure required by the system they are supplying. The relief valve prevents system and pump damage if the pump discharge valve is shut during pump operation or if any other occurrence such as a clogged strainer blocks system flow [11].

### 2.6 Linear Hydraulic Actuator

Linear actuators are the devices for converting fluid power into linear motion. They may be used to exert a force, to hold or clamp, and to initiate or stop motion. All linear actuators are some modification of an air or hydraulic cylinder and may be either single or double acting. The single acting cylinder receives power at one end only and is returned to its original position by gravity or by spring action, while double acting cylinder is powered in both directions. Double-acting cylinders permit more complete control of movement. Ram is a form of single acting cylinder in which the piston rods are of the same diameter. The
size should be large enough not only to handle the loads expected during duty cycle but also ensure the required load velocity. It is also important not to oversize actuators so that the flow required for maximum velocity is kept to a minimum. Otherwise, the hydraulic power supply becomes bulky with large no load power losses. Also, the load should be properly matched to the output of the system. Load matching effectively utilizes the output power of hydraulic source and improves performance of the hydraulic system.

The actuator of Rosaries Hydro turbine governor, which is apart of the subject of this thesis, is a single rode double acting type. A cross section and layout of the actuator is shown in figure 2.5.
Figure 4.5: Plant Linear Actuator (a) cross section (b) layout of the actuator.
2.7 Hydraulic Power Supply efficiency analysis

The following discussion covers the theoretical power losses in simple CP valve controlled hydraulic systems. For simplicity, the hydraulic actuator is assumed to be double rod with equal areas at each side of the piston and the hydraulic servo/proportional valve is assumed to be zero lapped. In a zero lapped valve, there is no dead band when the spool is centered. The orifice opening is zero for the centered spool position and under constant pressure drop across the valve the valve flow gain is constant for every spool position. The hydraulic circuit representation of such a system is shown in Figure 2.6.

![Figure 2.6: Constant Pressure Valve Controlled Actuator Circuit](image)

Figure 2.6: Constant Pressure Valve Controlled Actuator Circuit
In Figure 2.5 only two of the servo valve ports are open at any time since the valve is zero lapped [12]. When \(X_v > 0\) (extension of the hydraulic actuator), the pressurized oil from the supply passes trough orifice 2 to the hydraulic cylinder chamber A, and the oil in chamber B passes through the orifice 4 back to the oil reservoir. When \(X_v < 0\) (retraction of the hydraulic actuator), the pressurized oil coming from the supply passes through orifice 3 to the cylinder chamber B and the oil at chamber A passes through the orifice 1 back to the oil reservoir.

Because the actuator has a double rod with equal areas, the flow rates passing through the orifices 2 and 4 for the extension and 1 and 3 for the retraction will always be the same. Moreover, because the valve is symmetric the orifice resistances are also identical. Therefore, in this series circuit, the pressure drop at each orifice will be the same and can be expressed as:

\[
\Delta P = \frac{P_s - P_t - P_l}{2}
\]  

(2.1)

where,

\(P_s\) represents the supply pressure,

\(P_t\) represents the hydraulic tank pressure.

\(P_l\) represents the load pressure; that is, the pressure drop across the load.

The flow rate through a servo valve is proportional to the square root of the pressure drop across the port and the valve opening. The flow rate through the load \(q_L\) is defined as:

\[
q_L = C_a w_o x_v \sqrt{\frac{2\Delta P}{\rho}} = C_a w_o x_v \sqrt{\frac{2}{\rho} \left[ \frac{P_s - P_t - P_l}{2} \right]}
\]  

(2.2)
Where:

$c_d$ represents the orifice discharge coefficient.

$w_o$ represents the perimeter of the orifice.

$x_v$ represents the orifice opening which is same as the spool position.

$\rho$ represents the hydraulic oil density.

By taking the squares of each side and rearranging the Eq. (2.2), the expression for the load pressure is obtained as:

$$p_L = p_s - p_t - \left( \frac{\rho}{c_d w_o x_v^2} \right) q_L^2$$  \hspace{1cm} (2.3)

If equation (2.3) normalized, a normalized load pressure can be obtained as:

$$\overline{p}_L = 1 - \frac{q_L^2}{\lambda^2}$$  \hspace{1cm} (2.4)

Where:

$$\overline{p}_L = \frac{p_L}{p_s - p_t}$$ Represents normalized load pressure.

$$\overline{q} = \frac{q_L}{q_{max}}$$ Represents normalized load flow rate.

$$\lambda = \frac{x_v}{x_{v-max}}$$ Represents normalized valve spool opening.

Using equation (2.4), valve characteristic curves can be sketched for the constant pressure zero lapped valve controlled circuit, as shown in the figure below:
Figure 2.7 Valve Characteristic Curves for Different Valve Openings

In figure 2.7, the 1x1 area formed by the normalized flow and pressure axes represents the total power supplied to the system by the Hydraulic power supply. The area formed by drawing perpendicular lines from an arbitrary point A on the valve characteristic curve to the normalized pressure and flow axes represents the power transmitted to the load by the valve. According to that graph for the valve to transmit the maximum power to the load for maximum efficiency, the point A should be on the curve drawn for maximum normalized valve opening; where $\lambda=1$.

Note that, any drive characteristic curve should enclose a dynamic load locus completely in order to successfully handle a given operation. The dynamic load locus is defined as the complete boundary of the region of the $\bar{q}_l$, $\bar{p}_l$ plane that may be swept out by the load during its full cycle. A load locus curve for dummy load is shown in figure 2.7 below.
In Figure 2.8, the region covered by the drive curve but not by the load locus represents the uneconomical over design. For an efficient design, this load locus should be tangent to the drive curve at one or more points without yielding to any excessive points above the drive curve.

The point of tangency of a load locus and a valve drive curve is represented by point A in Figure 2.8. Now the problem is to determine the coordinates of point A which will represent the peak power requirement of the load is equal to the maximum power that can be transmitted by the valve. In other words, this point A will represent the maximum theoretical output power of an ideal constant pressure supply valve controlled circuit. This can be found by writing the normalized power equation.
transmitted to the load, which is the area formed by drawing perpendicular lines to the axis.

The power transmitted to the load is:

\[ \bar{P} = \bar{q}_L \bar{p}_L \]  \hspace{1cm} (2.5)

From the Eq. (2.4) for maximum spool opening $\lambda=1$, power transmitted to the load becomes:

\[ \bar{P} = \bar{q}_L (1 - \bar{q}_L^2) \]  \hspace{1cm} (2.6)

If the Eq. (2.6) differentiated with respect to normalized flow $\bar{q}_L$, and set zero, the normalized flow rate required for maximum power output is found as follows:

\[ \dot{\bar{P}} = 1 - 3\bar{q}_L^2 = 0 \]  \hspace{1cm} (2.7)

So: \[ \bar{q}_L = \sqrt{\frac{1}{3}} \]  \hspace{1cm} (2.8)

And from Eq. (2.4) the corresponding normalized pressure is found as:

\[ \bar{p}_L = \frac{2}{3} \]  \hspace{1cm} (2.9)

Hence the maximum theoretical normalized power output of the CP valve controlled system is found to be:

\[ \bar{P}_{\text{max}} = \bar{q}_L * \bar{p}_L = 0.385 \]  \hspace{1cm} (2.10)
That means at maximum only 38.5% of the total hydraulic power supply can be delivered to the load. The remaining power is lost on the pressure relief valve and the flow control valve. The excess flow rate of the pump which is equal to \( 1 - \bar{q}_t \), returns to the tank through the pressure relief valve, with a normalized pressure drop value of 1 across it, then the power loss on the pressure relief valve can be found as:

\[
\bar{P}_{Rv-loss} = 1 * \left( 1 - \frac{1}{\sqrt{3}} \right) = 0.423
\]  

(2.11)

The power loss on the flow control valve is equal to the multiplication of normalized load flow rate by the normalized pressure drop across the flow control valve, which can be defined as:

\[
\bar{P}_{Fv-loss} = \frac{1}{\sqrt{3}} * \left( 1 - \frac{2}{3} \right) = 0.192
\]  

(2.12)

All these losses are represented in Figure 2.7. Area 1 represents the maximum theoretical power that can be transmitted to the load. Area 2 represents the power loss on the relief valve and the area 3 shows the power loss on the flow control valve.

Note that all these calculations are carried out by assuming a dummy load whose peak power requirement is equal to the maximum power output of the series valve circuit. The analysis above is to find the efficiency for an instant of time.
corresponding to the maximum power requirement of the load. During the duty cycle of the load the efficiency of the hydraulic circuit will be less than 38.5%. For example, the load locus of the fictitious load in Figure 2.7 is tangent to the valve curve only at one point at A, that is in all remaining times of its duty cycle the valve opening ratio $\lambda$, will be smaller than 1 so that decreasing the overall efficiency.

Theoretically, if the pump flow rate delivered to the system is adjusted so that there is no excess flow over relief valve, then at point A the maximum power output of the system will be 66.7%.

Another source of the power loss is the throttle losses on the zero lapped flow control valve which corresponds to 19.2% of the total power supplied to the system, at the instant of maximum power output. The valve under analysis is a zero lapped 4-way valve which is modeled as a series circuit, where only two ports of the valve remain open at any instant of time. As these two ports are mechanically connected, their resistance to flow is the same for any spool movement. Thus, half of the power lost is on the meter-in port, which is the port where the flow coming from the supply pressure passes through the hydraulic cylinder chamber, and the remaining half of the power is lost on the meter-out port where the flow coming from the hydraulic cylinder chamber passes through the tank. By utilizing mechanically decoupled meter-in and meter-out valves, the power lost on the flow control valve can be decreased as their resistance will not have to be the same and adjusted independently.
Chapter 3
Mathematical Modeling

3.1 Hydraulic Actuator Model

As there are a lot of hydraulic actuator models in literature, the hydraulic cylinder model is given below without going in its details. The assumptions used to model the hydraulic cylinder are:

- The leakage coefficient between the two chambers of the hydraulic cylinder is laminar flow and it is proportional with the differential pressure between them.
- The friction force between the hydraulic cylinder and the piston sealing is assumed to be proportional with the cylinder velocity. Only viscous friction is included in the system non-linear model.
- The hydraulic piston is assumed to be a distinct load and lumped into the mass which is connected to the hydraulic cylinder.
- The chamber volumes are assumed to be constant in linear mathematical model. However in the MATLAB Simulink model, the chamber volumes are changing proportional to the cylinder position. In the hydraulic actuator model the hydraulic cylinder chamber A (cap-end) is assumed to be inlet and the hydraulic cylinder chamber B (rod-end) is assumed to be the outlet. Thus, the forward movement of the cylinder is assumed to be positive. In Figure 3-8, the positive flow rate \( q_A \) that is entering the chamber A, and the positive flow rate \( q_B \) that is leaving the chamber B are shown. The continuity equations for the hydraulic cylinder chambers can be written as:

\[
q_A = A_A \frac{d}{dt} \left( x + \frac{V_A}{E} \right) \quad \text{(3.1)}
\]
\[ q_B = A_B x \frac{V_B}{E} \frac{dp_B}{dt} \quad 3.2 \]

and the load pressure is defined as

\[ p_A A_A - p_B A_B = A_B (\gamma p_A - p_B) \quad 3.3 \]

\[ p_L = \gamma p_A - p_B \quad 3.4 \]

Where \( \gamma \) is piston annulus areas ratio, then, the force transmitted to the load will be expressed by the equation:

\[ f_L = p_L \cdot A_B \quad 3.5 \]

The MATLAB Simulink model of the hydraulic actuator is represented in Figure 3.1. The inputs to this sub-system are the flow rates of the inlet and outlet ports of the Electro hydraulic valve in terms of [mm3/s] and the outputs of the subsystem are the chamber A and chamber B pressures \( p_A, \ p_B \) in terms of [MPa] and the load force \( f_L \) in terms of [N].

In the MATLAB Simulink model of the system, the hydraulic cylinder chamber volumes are not constant but changing with the hydraulic cylinder position. In fact, this does not affect the simulation results much as the dead volume due to the transmission lines are much more than the volume change due to the cylinder position. Hydraulic cylinder chamber volume models in MATLAB Simulink environment are given in Figure 3.2. The common input of both subsystems is the hydraulic cylinder position, \( x \), in terms of [mm], and the outputs of the sub-systems are the chamber volumes \( VA, VB \) in terms of [mm3].
Figure 3.1: MATLAB Simulink Model of the Hydraulic Actuator

Figure 3.2: MATLAB Simulink Model of the Hydraulic Cylinder Chamber Volumes
3.2 Hydraulic Power Supply Model

The behavior of the actuator hydraulic power supply described in chapter 2, can be treated in the same way as the cylinder chambers pressures that means by applying the flow continuity equation to the volume of trapped oil between the pump and the electro hydraulic valve. In this case, the input flow rate is held constant by the constant speed of the pump motor, and the piping and accumulator assumed to be rigid. The transformed equation is:

\[ P_s = \frac{\beta}{V} \int (Q_{\text{pump}} - Q_L) \, dt \]  

This equation takes into account the load flow (QL) drawn from the supply through the proportional servo valve, and accurately models the case of a high actuator slew rate resulting in a load flow which exceeds the flow capacity of the pump. The action of the Loading/Unloading valve is modeled using a combinational logic to switch the pump volumetric flow rate at predefined cut-in and cut-out system pressure values. The Simulink model of the supply system is shown in figure 3.3.

Figure 3.3: Hydraulic Power Supply Simulink schematic
3.3 Load Model

The test system load can simply be thought as a mass-damper system. The mass consists of the hydraulic piston and the steel plate attached to it, and represented by $m$. The friction force which is assumed to be viscous constitutes the damping part of the load and the viscous friction coefficient is represented by $b$. The friction force acting on the load is highly non-linear. However to have a linear model, there assumed to be viscous friction between the hydraulic cylinder and piston sealing. The friction is not a parameter that can be measured directly or specified by manufacturer. In this thesis, the friction characteristics of the hydraulic cylinder are determined through an experimental procedure by measuring the hydraulic cylinder chamber pressures. After modeling the system as a mass-damper system, the structural equation for the load by using the Newton’s 2nd law, can be written as:

\[ f_L = m \ddot{x} + b \dot{x} \]  

3.7
3.4 Valve Controlled System

Parts of the pilot actuated electro hydraulic proportional valve controlled system model are derived depending on the manufacturer’s frequency response test data, for example the transfer function of the force motor controlled pilot stage which will be detailed in the next sections.

3.4.1 Mathematical Modeling of the System

The pilot valve position is controlled via digital governor, which supplies a DC voltage in the range of (-10: 0: +10), the position of the main valve spool is summed mechanically with the pilot stage spool position. The bandwidth of the valve for 100% command input signal is around 59.4 Hz. which is very high with respect to the hydraulic applications, and can be assumed to be an ideal flow rate source for a given reference spool position command. Nevertheless, the valve dynamics are modeled in order to investigate the dynamic load response of the actuator, which will be presented in the next chapters.

3.4.1.1 Electro Hydraulic Transducer Transfer function

A servo-valve is a complex device which exhibits a high-order non-linear response, and knowledge of a large number of internal valve parameters is required to formulate an accurate mathematical model. Indeed, many parameters such as orifice sizes, spring rates, and spool geometry and so on, are adjusted by the manufacturer to tune the valve response and are not normally available to the user. It is sometimes possible to ignore any inherent non-linearities and employ a small perturbation analysis to derive a linear model which approximates the physical system. Such models are often based on classical first or second order differential equations, the coefficients of which are chosen to
match the response of the valve based on frequency plots taken from the data sheet.

A simple second order model yields only an approximation to actual behavior, however the servo-valve is not the primary dynamic element in a typical hydraulic servo system and is generally selected such that the frequency of the 90 degree phase point is a factor of at least three higher than that of the actuator [10]. For this reason it is usually only necessary to accurately model valve response through a relatively low range of frequencies, and the servo-valve dynamics may be approximated by a second order transfer function without serious loss of accuracy.

![Frequency response of the TR-h7/80F 0.6](image)

Figure 3.4: Frequency response of the TR-h7/80F 0.6

A performance graph of the servo valve is shown in Figure 3.4. Assuming a second order approximation is to be used; suitable values for natural frequency and damping ratio will need to be determined from the graph. Natural frequency ($\omega_n$) can be read fairly accurately from the 90 degree phase point, which is 59.4.
Damping (\(\xi\)) can be determined from an estimate of the magnitude of the peaking present. For an under-damped second order system, the damping factor (\(\xi\)) can be shown to be related to peak amplitude ratio (\(M\)) by the formula:

\[
M = e^{\frac{-2\xi}{\sqrt{1-\xi^2}}}
\]  

3.8

A reasonable estimate of peaking based on performance graph of the servo valve would be about 1.283. A suitable value of damping determined iteratively from Eq.3.8 is about 0.433.

So, the force motor controlled pilot stage transfer function is represented as:

\[
\frac{X(s)}{V(s)} = \frac{0.055099}{0.00028341\ 77918\ s^2 + 0.01582491\ 5\ s + 1}
\]  

3.9

3.4.1.2 The main flow valve model

The main valve used in Rosaries’ actuators is a servo proportional close centered zero-lap valve therefore, as shown in figure 3.5 there is no dead zone or initial opening, the valve orifice area is proportional to the spool displacement at any time. Thus, under constant pressure difference across the valve, the flow gain is constant and does not change with the spool position. The constant pressure difference across the valve is guaranteed by modeling an accumulator for constant pressure supply.
In the zero-lap valve, only two of the arms numbered 1, 2, 3, and 4 are open at any time; therefore only two orifice equations can represent the valve dynamics for either extension or retraction. Assuming zero tank pressure, these expressions can be written as follows. For extension case, positive spool position $X_v > 0$

$$q_2 = C_d w_0 x_v \sqrt{\frac{2}{\rho} \left( p_s - p_A \right)}$$  \hspace{1cm} 3.10

$$q_4 = C_d w_0 x_v \sqrt{\frac{2}{\rho} p_B}$$  \hspace{1cm} 3.11

For retraction case, negative spool position $x_v < 0$;

$$q_1 = C_d w_0 x_v \sqrt{\frac{2}{\rho} p_A}$$  \hspace{1cm} 3.12
\[ q_3 = C_d w_0 x_v \sqrt{\frac{2}{\rho} (p_s - p_B)} \]  

3.13

Note that the valve and parameters discharge coefficient \( C_d \), orifice area gradient (w), and \( \rho \) is the hydraulic oil density. are constants and generally not given in the manuals. Instead, they are represented by a flow gain \( K_v \), that can be obtained from the valve manual from the relation between the flow rate and valve input current:

\[ K_v = C_d w_0 \sqrt{\frac{2}{\rho}} \]  

3.14

Substituting plugging \( k_v \) in the orifice flow equations above, we obtain the following meaningful form:

\[
\begin{align*}
q_4 &= k_v x_v \sqrt{p_B} \\
q_2 &= k_v x_v \sqrt{(p_s - p_A)} \\
q_1 &= k_v x_v \sqrt{p_A} \\
q_3 &= k_v x_v \sqrt{(p_s - p_B)}
\end{align*}
\]

3.15

for \( x_v > 0 \)

for \( x_v < 0 \)

The MATLAB Simulink model of the valve is shown in Figure 3.6. The presented portion is for port A of the valve. Here the input to valve sub-system is the spool position in (mm) and the output is the flow rate to the asymmetric cylinder chambers ports in (m3/s). The constant \( K \) is the valve flow gain.
3.4.2 Steady State Characteristics of the System

An asymmetric cylinder has different characteristics for extension stroke than retraction; this is mainly due to difference between rode side and cap side areas for the piston annulus.

At steady state the cylinder chambers pressure change converges to zero, so the flow continuity equations in 3.1 and 3.2 can be expressed for both extension and retraction cases as follows:

\[ q_{B\_ss} = A_B \dot{x} \]  
\[ q_{A\_ss} = A_A \dot{x} = \gamma A_B \dot{x} \]
So the steady state relation between the flow through into chamber A and through out chamber B is:

\[ q_{A\_ss} = \gamma q_{B\_ss} \]  \hspace{1cm} (3.18)

So, by substituting in the valve flow equations 3.15, the steady state relation chambers pressure can be found as;

\[ q_{A\_ss} = q_2 = k \frac{x}{v} \sqrt{p_s - p_{A\_ss}} = \gamma k \frac{x}{v} \sqrt{p_{B\_ss}} = \gamma q_4 = \gamma q_{B\_ss} \]  \hspace{1cm} (3.19)

\[ p_{A\_ss}^2 + \gamma^2 p_{B\_ss}^2 = p_s^2 \]  \hspace{1cm} (3.20)

For extension case, and for retraction case;

\[ q_{A\_ss} = q_1 = k \frac{x}{v} \sqrt{p_{A\_ss}} = \gamma k \frac{x}{v} \sqrt{p_{s} - p_{B\_ss}} = \gamma q_3 = \gamma q_{B\_ss} \]  \hspace{1cm} (3.21)

\[ p_{A\_ss}^2 + \gamma^2 p_{B\_ss}^2 = \gamma^2 p_s^2 \]  \hspace{1cm} (3.22)

When zero load pressure;

\[ p_{A\_ss} + \gamma p_{B\_ss} = 0 \]  \hspace{1cm} (3.23)

Hence steady state chambers pressure can be written in terms of supply pressure as, for extension case;

\[ p_{A\_ss\_ext} = \frac{p_s}{\gamma^3 + 1} \]  \hspace{1cm} (3.24)
\[ p_{B_{ss\_ext}} = \frac{\gamma p_s}{\gamma^3 + 1} \] 3.25

And for retraction case;

\[ p_{A_{ss\_ext}} = \frac{\gamma^2 p_s}{\gamma^3 + 1} \] 3.26

\[ p_{B_{ss\_ext}} = \frac{\gamma^3 p_s}{\gamma^3 + 1} \] 3.27

### 3.4.3 Linearized Valve Coefficients

As shown in the above equations the valve flow rates are highly non-linear, in order to investigate the dynamic loading behavior of the system a linearization shall be made to the characteristic valve flow equation so as to get a linear relationship between servo-valve spool position and actuator piston position.

To do so, the chamber pressures under dynamic loading are assumed to be approaching the steady state cylinder chamber pressures. In other words, the load pressure change is neglected relative to the chambers pressure.

Then the flow continuity equations defined by Eq. (3.15) can be linearized at the steady state pressures defined by Eq. (3.24) through Eq. (3.27) for a given constant reference spool position \( x_v \).
3.4.3.1 Extension Case

For the extension case figure 3.7, the pressurized oil coming from the supply passes through the orifice 2 and goes to the chamber A and the oil in chamber B passes through orifice 4 and goes to the tank. Therefore, for the extension case, the linearization of the orifices 2 and 4 for a given spool input position \( x_{s0} \) at steady state extension chamber pressures \( p_{ext} \) and \( p_{ext} \) should be performed.

Figure 3.7: Convention of valve ports flow (extension).

**Orifice 2:**

The flow rate passing through the orifice 2 can be linearized as follows,

\[
q_2 = k_v x_v \sqrt{p_s - p_{A_{ss_{-ext}}}} = K_{u2_{-ext}} x_v - K_{p2_{-ext}} p_A
\]

Here the terms \( K_{x_{v2_{-ext}}} \) is valve spool position gain of orifice 2 linearized at the spool position \( x_{v0} \) and steady state chamber pressure \( p_{A_{ss_{-ext}}} \).

\[
K_{u2_{-ext}} \frac{\partial q_2}{\partial x_v} \bigg|_{x_v = x_0} = k_v \sqrt{p_s - p_{A_{ss_{-ext}}}} = k_v \sqrt{p_s - \frac{p_s}{\gamma^3 + 1}}
\]

\[
K_{u2_{-ext}} = K \frac{\gamma^3 p_s}{\gamma^3 + 1}
\]

3.28

3.29
The term $K_{p2_{\text{ext}}}$ is the valve pressure gain of orifice 2 which is also linearized at the spool position $x_{v0}$, and steady state chamber pressure $p_{A_{ss_{\text{ext}}}}$ as follow:

$$K_{p2_{\text{ext}}} = \frac{\partial q_2}{\partial p_A} \bigg|_{x_v = x_0} = \frac{k_v \cdot x_{v0}}{2 \sqrt{p_s - p_{A_{ss_{\text{ext}}}}}} = \frac{k_v \cdot x_{v0}}{2 \sqrt{p_s - \frac{p_s}{\gamma^3 + 1}}}$$

$$K_{p2_{\text{ext}}} = \frac{k_v \cdot x_{v0}}{2 \sqrt{\frac{\gamma^3 p_s}{\gamma^3 + 1}}}$$  \hspace{1cm} 3.30

**Orifice 4:**

The flow rate passing through the orifice 4 is linearized as:

$$q_4 = k_v \cdot x \sqrt{p_B} = K_{u4_{\text{ext}}} x_{v0} - K_{p4_{\text{ext}}} p_s$$  \hspace{1cm} 3.31

Here the terms $K_{x_{v4_{\text{ext}}}}$ is valve spool position gain of orifice 4 linearized at the spool position $x_{v0}$ and steady state chamber pressure $p_{B_{ss_{\text{ext}}}}$.

$$K_{u4_{\text{ext}}} = \frac{\partial q_4}{\partial x_v} \bigg|_{x_v = x_0} = k_v \sqrt{p_{B_{ss_{\text{ext}}}}}$$

$$K_{u4_{\text{ext}}} = \frac{\gamma p_s}{\gamma^3 + 1}$$

$$K_{u4_{\text{ext}}} = K_v \sqrt{\frac{\gamma p_s}{\gamma^3 + 1}}$$  \hspace{1cm} 3.32
The term $K_{p4\_ext}$, is the valve pressure gain of orifice 4, which is also linearized at the spool position $x_v0$ and steady state chamber pressure $p_{B\_ss\_ext}$.

\[
K_{p4\_ext} = \frac{\partial q_4}{\partial p_B} \bigg|_{x_v = x_0} = \frac{k_{v\_v0}}{2 \sqrt{p_{B\_ss\_ext}}}
\]

\[
K_{p4\_ext} = \frac{k_{v\_v0}}{2 \sqrt{\gamma p_s}} \frac{\gamma^3 + 1}{\gamma^3 + 1}
\]

3.33

Note that the valve spool position gain of the orifice 2 is $\gamma$ times the valve spool position gain of orifice 4.

\[
K_{u2\_ext} = \gamma K_{u4\_ext}
\]

3.34

The valve pressure gain of the orifice 4 is $\gamma$ times the valve pressure gain of orifice 2.

\[
K_{p2\_ext} = \frac{K_{p4\_ext}}{\gamma}
\]

3.35

3.4.3.2 Retraction Case

The retraction case is shown in figure 3.8, during which the oil coming from a pressurized source passes through the orifice 3 and start work on chamber B, in the other side oil exits out of chamber A passes through orifice 1 and goes to the tank.
The next section, discusses the linearization procedures of orifices 1 and 3 for input spool position $x_{v0}$ for a retraction cycle at steady state chamber pressure $p_{A_{ss\_ret}}$, and $p_{B_{ss\_ret}}$.

**Orifice 3:**

The flow rate passing through the orifice 3 can be linearized as follows:

$$q_3 = k_v x_v \sqrt{p_s - p_B} = K_{u3\_ret} x_v - K_{p3\_ret} p_B$$  \hspace{1cm} 3.36

Here the terms $K_{xv3\_ret}$ is valve spool position gain of orifice 3 linearized at the spool position $x_{v0}$ and steady state chamber pressure $p_{B_{ss\_ret}}$.

$$K_{u3\_ret} = \frac{\partial q_3}{\partial x_v} \bigg|_{x_v = x_{v0}} = k_v \sqrt{p_s - p_{B_{ss\_ext}}}$$  \hspace{1cm} 3.37

$$K_{u3\_ret} = k_v \sqrt{\frac{p_s}{\gamma^3 + 1}}$$

The term $K_{p3\_ret}$, is the valve pressure gain of orifice 3, which is also linearized at the spool position $x_{v0}$ and steady state chamber pressure $p_{B_{ss\_ret}}$.

$$K_{p3\_ret} = \frac{\partial q_3}{\partial p_B} \bigg|_{x_v = x_{v0}} = \frac{k_v x_{v0}}{2 \sqrt{p_s - p_{B_{ss\_ext}}}}$$  \hspace{1cm} 3.38

$$K_{p3\_ret} = \frac{k_v x_{v0}}{2 \sqrt{\gamma p_s \left(\gamma^3 + 1\right)}}$$
Orifice 1:

The flow rate passing through the orifice 1 can be linearized as follows:

$$q_1 = k \cdot \sqrt[3]{p_A} = K_{u1\_ret} x_v - K_{p1\_ret} p_A$$  \hspace{1cm} 3.39

Here the terms $K_{x_{v1\_ret}}$ is valve spool position gain of orifice 1 linearized at the spool position $x_v0$ and steady state chamber pressure $p_{A\_ss\_ret}$.

$$K_{u1\_ret} = \frac{\partial q_1}{\partial x_v} \bigg|_{x_v = x_0} = k \cdot \sqrt[3]{p_{A\_ss\_ret}}$$  \hspace{1cm} 3.40

$$K_{u1\_ret} = K_v \sqrt{\frac{\rho_s^2}{\gamma^3 + 1}}$$
The term $K_{p1\_ret}$, is the valve pressure gain of orifice 1, which is also linearized at the spool position $x_{v0}$ and steady state chamber pressure $p_{A\_ss\_ret}$.

$$K_{p1\_ret} = \frac{\partial q_3}{\partial p} \bigg|_{x_v = x_0}^{p_A = p_{A\_ss}} = \frac{k_{v\cdot x_v0}}{2 \sqrt{p_{A\_ss\_ret}}} = \frac{k_{v\cdot x_v0}}{2 \sqrt{\gamma^2 p_s}}$$

$$K_{p1\_ret} = \frac{k_{v\cdot x_v0}}{2 \sqrt{\frac{\gamma^2 p_s}{\gamma^3 + 1}}}$$

3.41

Note that the valve spool position gain of the orifice 1 is $\gamma$ times the valve spool position gain of orifice 3.

$$K_{u1\_ret} = \gamma K_{u3\_ret}$$

3.42

The valve pressure gain of the orifice 3 is $\gamma$ times the valve pressure gain of orifice 1.

$$K_{p1\_ret} = \frac{K_{p3\_ret}}{\gamma}$$

3.43
3.4.4 Dynamic Characteristics of the System

In this section, a transfer function between the input valve spool position and the output cylinder rod velocity is derived. In order to obtain a linear relationship, the linearized valve flow coefficients found in the previous subsection are to be used. A dynamic analysis for the extending case is carried out below. Since the procedure is the same; the transfer function derivation for the retraction case is not explained. Two flow continuity equations of the cylinder chambers and valve and one structural equation of the load define the system dynamics.

For the cap end of the hydraulic cylinder, the flow continuity equation can be written by using the linearized valve flow equation (3.28), and the flow continuity equation of the cylinder chamber equation (3.1):

\[ q_2 = q_A \]

\[ k u_2_{\text{ext}} \dot{x} - k p_2_{\text{ext}} p_A = A_A \ddot{x} + \frac{V_A}{E} \frac{dp_A}{dt} \]  

3.44

For the rod end of the hydraulic cylinder, the flow continuity equation can be written by using the linearized valve flow equation Eq.(3.2) and the flow continuity equation of the cylinder chamber Eq.(3.31):

\[ q_4 = q_B \]

\[ k u_4_{\text{ext}} \dot{x} - k p_4_{\text{ext}} p_B = A_B \ddot{x} + \frac{V_B}{E} \frac{dp_B}{dt} \]  

3.45

The structural equation of the load is the valve controlled actuator given by Eq. (3. 7) and it is repeated here as;
These 3 equations, with one known control input $x_v$, and three unknowns, $p_A$, $p_B$ and cylinder rod velocity $\dot{x} = v$, can be solved to find the transfer function between the input spool position $x_v$ and cylinder velocity $\dot{x} = v$. The derivation of the transfer function is detailed in appendix A.

The transfer function between the reference input spool position $x_v$ is as follow:

$$V(s) = \frac{a_1 s + a_2}{X(s)} = \frac{a_1 s + a_2}{b_1 s^3 + b_2 s^2 + b_3 s + b_4}$$

$$a_1 = k_{u4_ext} A_b (\gamma^2 + \alpha) \frac{V_b}{E}$$

$$a_2 = k_{u4_ext} A_b (\gamma^3 + 1) k_{p2_ext}$$

$$b_1 = m \frac{\alpha V^2_b}{E^2}$$

$$b_2 = m k_{p2_ext} \frac{V_b}{E} (\gamma \alpha + 1) + b \frac{\alpha V^2_b}{E^2}$$

$$b_3 = m \gamma k_{p2_ext}^2 + b k_{p2_ext} \frac{V_b}{E} (\gamma \alpha + 1) + (\gamma^2 + \alpha) \frac{V_b}{E} A_B^2$$

$$b_4 = b \gamma k_{p2_ext}^2 + (\gamma^3 + 1) k_{p2_ext} A_B^2$$

The result is a 3rd order transfer function. Since the characteristic equation cannot be written in a factored form; it is very hard to interpret how the system parameters affect the roots of the characteristic equation. Therefore a relationship between the chamber pressures will be defined to reduce the order of the system.
By using Eq.(3.34), Eq.(3.35), Eq. (3.44) and Eq. (3.45) the relation between the chamber pressures can be written. Inserting Eq.(3.34) into Eq. (3.44), Eq.(3.35) into Eq. (3.45), multiplying Eq. (3.45) by \( \gamma \) and subtracting from Eq. (3.44) the relation between \( p_A \) and \( p_B \) in s-domain can be obtained as follows:

\[
P_A(s) = -\frac{\gamma \frac{V_B}{E} s + \gamma^2 k_{p2_{\text{ext}}}}{\frac{V_B}{E} s + k_{p2_{\text{ext}}}} P_B(s)
\]

3.48

This equation represents the dynamic pressure changes under an applied load; from the frequency response plot presented in figure 3.9 it’s clear that the relation between chambers pressure change is linear above and below some predetermined frequencies, in case of rapid load pressure changes the relation can between chambers pressure change will be:

\[
P_A(s) = -\frac{\gamma V_B}{V_A} P_B(s)
\]

3.49

And in case of slowly changing load pressure the relation is reduced to:

\[
P_A(s) = -\gamma^2 P_B(s)
\]

3.50
Figure 3.9: Chambers Pressure Ratios

As shown in figure 3.9, at low excitation frequencies that the dynamic pressure ratio of the cylinder chambers is 1.637dB (magnitude of 1.207), which is equal to $\gamma^2$. At higher frequencies larger than 1 Hz, the dynamic pressure change ratio drops to 0.5474 dB (magnitude of 1.065), which is equal to the value of $\gamma V_B/V_A$. Practically, this means that under an oscillatory dynamic loading whose frequency is higher than 1 Hz, to compensate the dynamic load pressure, the chamber B pressure will reduce $\Delta p$ from its steady state value, while chamber A pressure increase $1.065\Delta p$ from its steady state value, so the order of the system reduces by one cause the chamber pressures now linearly dependent. The reduced transfer function is:

$$V(s) = \frac{(\gamma^2 + \alpha)k_{uA_{ext}}A_B}{\alpha \gamma B E^2 s^2 + \left( m \frac{\phi + \alpha}{\gamma \phi + 1} \gamma k_{p2_{ext}} + \frac{b \alpha \gamma B E}{\gamma \phi + 1} \gamma k_{p2_{ext}} + (\gamma^2 + \alpha)A_B \right)}$$

3.51
In Figure 3.10 below the equivalent block diagram representation of this reduced order asymmetric cylinder proportional valve controlled actuator of the roseires turbine speed governor is given for the extension case, here \( V \) is the velocity of the piston. Note that, for the retraction case, the following replacements for the linearized valve spool position and valve pressure coefficients should be made:

\[
K_{u4_{\text{ext}}} \rightarrow K_{u3_{\text{ext}}}
\]

\[
k_{p2_{\text{ext}}} \rightarrow k_{p1_{\text{ext}}}
\]

![Figure 3.10: Block diagram of the EHA](image)
This second order transfer function can be used to understand the dynamic behavior of the system. The natural frequency and the damping ratio of the Actuator can be written as:

\[
\omega_n = \sqrt{\frac{b \frac{\varphi + \alpha}{\gamma \varphi + 1} \gamma K_{p2_{\text{ext}}} + (\gamma^2 + \alpha)A_B^2}{m \alpha V_B}}
\]

\[
\xi = \frac{1}{2} \frac{\sqrt{E\left(b \frac{\varphi + \alpha}{\gamma \varphi + 1} \gamma K_{p2_{\text{ext}}}ight) + m \frac{\varphi + \alpha}{\gamma \varphi + 1} \gamma K_{p2_{\text{ext}}}}}{m \alpha V_B \left(b \frac{\varphi + \alpha}{\gamma \varphi + 1} \gamma K_{p2_{\text{ext}}} + (\gamma^2 + \alpha)A_B^2\right)}
\]

Depending on the spool position where the linearization is performed, as the valve pressure gain decreases with the increasing supply pressure, it seems that the natural frequency of the open loop system will decrease with the increasing supply pressure. However it should be noted that as the valve spool position gain also depends on the supply pressure, the response of the closed loop system will increase by increasing the supply pressure as it will increase the valve spool position gain which is the open loop gain and shown in Figure 3.10.

Other important parameters which determine the natural frequency of the system is the hydraulic cylinder chamber volumes, bulk modulus of the hydraulic oil and cylinder area. The natural frequency of the system increases with the cylinder area and bulk modulus of the oil, whereas decreases with the hydraulic cylinder volume. Furthermore, the load mass decreases the natural frequency of the system as expected. Lastly, the term \((\gamma^2 + \alpha)\) appearing in the above equations indicate that increasing the area ratio and dead volume ratio, increases the natural frequency of the system while decreases the damping ratio.
Lastly the linear dynamic chamber pressure change assumption is checked. Table 3.1 gives the roots of the characteristic equations of the reduced second order transfer function defined by Eq. (3.51), and the third order transfer function defined by Eq. (3.47). The numerical values of the system parameters for the calculation of the transfer functions are taken from Table 3.2 and the valve flow coefficients are linearized at the spool position \( x_0 = 0.012 \text{mm} \) for supply pressure \( P_s = 3.5 \text{ MPa} \).

<table>
<thead>
<tr>
<th>Category</th>
<th>Poles</th>
<th>Zeros</th>
<th>Error between The poles of 3rd order TF and 2nd order TF</th>
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<td>-0.1373</td>
<td>-</td>
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<tr>
<td>2nd order TF</td>
<td>( \phi = 1.065 )</td>
<td>0</td>
<td>0.00+0.02i</td>
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<td>2nd order TF</td>
<td>( \phi = 1.208 )</td>
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<td>( \phi = 2 )</td>
<td>0</td>
<td>0.03+0.02i</td>
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As expected, it is seen in Table 3.1 that third pole and zero of the general 3rd order transfer function are very close, canceling each other, the remaining complex conjugate pole pairs are very close to the pole pair of the reduced second order system. Furthermore, the error between the real third order transfer function poles and second order transfer function poles are much smaller if the dynamic chamber pressure change ratio, \( \phi \) is determined for higher excitation frequencies.
### Table 3.2 Numerical Values of the System Parameters

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<tr>
<th><strong>Hydraulic Cylinder Parameters</strong></th>
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<td>mm²/(s.v)</td>
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</table>
4.1 State Space Representation

In the previous chapter, it was mentioned that two non-linearity sources are inherently exist with in single rod asymmetric actuator, one is the pressure flow relationship defined by Eq. (3.1). This non-linear flow equation is linearized around steady state chamber pressures and a prescribed spool position. The other source of non-linearity is the result of the single rod cylinder with unequal piston areas, this result in unequal flow gains for the retracting and extraction of the hydraulic circuit. As a result, a piecewise linearized system is formed, the linearized dynamic equations assuming linear steady state relation between cylinder chambers pressure are written both for extension and retraction cases.

As stated above, the order of the system can be reduced by assuming a linear relationship between the dynamic pressure changes of the hydraulic cylinder chambers and using dynamic load pressure $PL$ instead of hydraulic cylinder chamber pressures $PA$ and $PB$.

\[
PA = -\varphi PB \tag{4.1}
\]
\[
P_L = \gamma PA - PB \tag{4.2}
\]

Then the states of the system will be:

$X1 = x$  cylinder Piston position.

$X2 = \dot{x}$  cylinder Piston velocity.

$X3 = PL$  Load pressure. \tag{4.3}
The corresponding state equations can be written if the assumed chamber pressure relations defined by Eq. (4.1) and Eq. (4.2) are substituted in the general form of state equations.

\[\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{b}{m} x_2 + \frac{A B}{m} x_3 \\
\dot{x}_3 &= -\left(\gamma^2 + \alpha\right) A B \frac{E}{\alpha V_B} x_2 - \frac{(\varphi + \alpha)}{\gamma \varphi + 1} K_{p2_{-ext}} \frac{E}{\gamma \varphi + 1} x_3 + (\gamma^2 + \alpha) K_{x4_{-ext}} \frac{E}{\alpha V_B} u
\end{align*}\]  

The state equations and the output expressions for the extension case of the hydraulic cylinder can be written in a matrix form as:

\[\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\
0 & -\frac{b}{m} & \frac{A B}{m} \\
0 & -\frac{(\gamma^2 + \alpha) A B}{\alpha V_B} & -\gamma K_{p2_{-ext}} \frac{(\varphi + \alpha)}{\gamma \varphi + 1} \frac{E}{\alpha V_B}
\end{bmatrix} \begin{bmatrix} x_1 \\
x_2 \\
x_3
\end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
(\gamma^2 + \alpha) K_{x4_{-ext}} \frac{E}{\alpha V_B}
\end{bmatrix} u
\]

Note that for the retraction case the pressure flow gain in the above state equations, \(K_{p2_{-ext}}\) should be replaced with \(K_{p1_{-ret}}\) and the linearized flow gain \(K_{x4_{-ext}}\) should be replaced by \(K_{x3_{-ret}}\).
4.2 State Feedback Gain controller Design

Because of the inherent property that different extending and retracting dynamic characteristics of the single rod cylinder, two set of controller gains are calculated one set for extension and another for retraction. The valve control system is designed using the linearized set of reduced order system equations defined in Section 4.1. through pole placement via linear state feedback. The system is defined by three states which are;

- cylinder position,
- cylinder velocity,
- load pressure.

The block diagram representation of the closed loop position control of the valve controlled system with the defined states is given in figure (4.1).

---

Figure 4.1: Block Diagram Representation of the Closed Loop EHA.
In Figure 4.1, the parameters (Kpos), (Kval), and (Kpl) represent the state feedback gains of the position velocity, and load pressure measurements.

Note that this block diagram representation is for the extension of the hydraulic actuator, for the retraction it will be the same if the replacement of valve gains defined in Eq. (3.52) are made.

After adding state feedback the closed loop transfer function of the position control system becomes:

\[
\begin{align*}
V(s) &= \frac{(\gamma^2 + \alpha)K_{x4,ext}A_BK_{pos}}{X(s)} \\
&= \frac{(\gamma^2 + \alpha)K_{x4,ext}A_BK_{pos}}{a_1s^3 + a_2s^2 + a_3s + a_4}
\end{align*}
\]

\[
a_1 = \frac{mcV_B}{E}
\]

\[
a_2 = m\left(\frac{\phi + \alpha}{\gamma \phi + 1}K_{p2,ext} + K_{pL}(\gamma^2 + \alpha)K_{x4,ext}\right) + b\frac{cV_B}{E}
\]

\[
a_3 = b\left(\frac{\phi + \alpha}{\gamma \phi + 1}K_{p2,ext} + K_{pL}(\gamma^2 + \alpha)K_{x4,ext}\right) + (\gamma^2 + \alpha)(A_B + K_{vel}K_{x4,ext})A_B
\]

\[
a_4 = (\gamma^2 + \alpha)K_{x4,ext}A_BK_{pos}
\]

In the controller designs, it is assumed that all the state variables are available for feedback. The state equations derived in previous section are used here; for the extension case:
For retraction:

\[
\begin{bmatrix}
    \dot{x}_{1_{\text{ret}}} \\
    \dot{x}_{2_{\text{ret}}} \\
    \dot{x}_{3_{\text{ret}}}
\end{bmatrix} = \begin{bmatrix}
    0 & 1 & 0 \\
    0 & -\frac{b}{m} & \frac{A_B}{m} \\
    0 & \frac{(\gamma^2 + \alpha)A_B}{\alpha V_B'} & -\gamma K_{p2_{\text{ext}}} \frac{(\phi + \alpha)}{\gamma \phi + 1} \frac{E}{\alpha V_B'}
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    (\gamma^2 + \alpha)k_{x_{4_{\text{ext}}}}E
\end{bmatrix} \frac{u_{\text{ext}}}{\alpha V_B'}
\]

\[y = \begin{bmatrix}1 & 0 & 0\end{bmatrix} \begin{bmatrix}x_1 \\
    x_2 \\
    x_3\end{bmatrix}\]

In the state feedback control algorithm of the valve controlled system, two different control signals are generated, one for extension and another for retraction.

\[u_{\text{ext}} = -k_{\text{ext}} x_{\text{in}}\]
\[u_{\text{ret}} = -k_{\text{ret}} x_{\text{in}}\]
Where $k_{ext}$, and $k_{ret}$ are state feedback gain vectors for extension, and respectively. All the closed loop poles of the system can be replaced at any arbitrary locations in the complex plane if the system is fully state controllable, requiring that the rank of the controllability matrix $M$, is equal to number of states, that is 3. The controllability matrix is defined by:

$$M = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$  \hspace{1cm} 4.12

Since $M$ is a 3x3 square matrix, the controllability condition reduces to

$$\det(M) = \left( \frac{(y^2 + \alpha)K_{x4,ext}E}{\alpha V_B} \right)^3 \left( \frac{A_B}{m} \right)^2 \neq 0$$  \hspace{1cm} 4.13

which is automatically satisfied, indicating that the system is fully state controllable. The valve system is linearized at a spool position corresponding to 1.4mm and for a supply pressure of 3.5 MPa. The numerical values of $A$, $B$, $M$ and $\det M$ are given below by using the numerical values of the hydraulic system parameters defined in table 3.2.

$$A_{ext} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 89294 \\ 0 & -2 & -14 \end{bmatrix}$$

$$A_{ret} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 89294 \\ 0 & -2 & -14 \end{bmatrix}$$
The characteristic equation of the system is obtained as for extension

$$|sI - A_{ext}| = s^3 + 10s^2 + 193890s + 0$$  \hspace{1cm} 4.14

with the following coefficients of the characteristic equation:

$$a_{1ext} = 10; \ a_{2ext} = 193890; \ a_{3ext} = 0$$

and for retraction the characteristic equation is;

$$|sI - A_{ret}| = s^3 + 10s^2 + 193890s + 0$$  \hspace{1cm} 4.16
with the following coefficients of characteristic equation:

\[ a_{1_{ret}} = 10; \quad a_{2_{ret}} = 193890; \quad a_{3_{ret}} = 0 \]

It is seen that since there is a free \( s \) term in the characteristic equation, the open loop system for position output behaves as an integrator. For the speed output system, the order reduces to two and the system is stable, as all the coefficients are of the same sign (positive). In this thesis study, the performance of the system is determined using a sine sweep test, therefore in order to match model with the non-linear system the state feedback gains are calculated for bandwidth of 2Hz according to frequency limits of the real plant then the non-linear system is tested for these gains and compared to the mathematical model.

For the closed loop position control system, in order to have a 2 Hz bandwidth, the desired closed loop poles are chosen as:

\[ \mu_1 = -2(2\pi); \quad \mu_2 = -700; \quad \mu_3 = -800 \]

The locations of the last two of the desired poles are chosen far away from the origin compared to the location of the first pole, which is the dominant one. The last two poles will decay very quickly, so that the first pole, closer to the origin, will dominate in system response and determine the band-width of the system. As a result the desired characteristic equation becomes:

\[(s - \mu_1)(s - \mu_2)(s - \mu_3) = s^3 + 1500s^2 + 578800s + 7033600 \quad 4.17\]
yielding the following coefficients of the desired characteristic equation,

\[ b_1 = 1500; \quad b_2 = 578800; \quad b_3 = 7033600 \]

Then the state feedback matrices set for extension and retraction can be found as:

\[
K_{\text{ext}} = \begin{bmatrix} b_3 - a_{3\text{ext}} & b_2 - a_{2\text{ext}} & b_1 - a_{1\text{ext}} \end{bmatrix} T^{-1}_{\text{ext}}
\]

\[
K_{\text{ret}} = \begin{bmatrix} b_3 - a_{3\text{ret}} & b_2 - a_{2\text{ret}} & b_1 - a_{1\text{ret}} \end{bmatrix} T^{-1}_{\text{ret}}
\]

\[
T = MW
\]

where M is the controllability matrix derived previously, and W is given by

\[
W_{\text{ext}} = \begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 193890 & 10 & 1 \\ 10 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}
\]

\[
W_{\text{ret}} = \begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 193890 & 10 & 1 \\ 10 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}
\]

thus T is calculated to be

\[
T_{\text{ext}} = \begin{bmatrix} 1190200 & 0 & 0 \\ -420000 & 1190200 & 0 \\ -500 & 400 & 100 \end{bmatrix}
\]

\[
T_{\text{ret}} = \begin{bmatrix} 1135400 & 0 & 0 \\ -288000 & 1135400 & 0 \\ 300 & 400 & 100 \end{bmatrix}
\]
where the transformation matrix $T$ is given by

Finally the desired feedback gain vector sets $K_{ext}$ and $K_{ret}$ are obtained by use of the Eq.(4.18), is calculated to be,

$$K_{ext} = [1.2028 \: 0.0546 \: 18.6307]$$

$$K_{ret} = [1.2967 \: 0.0574 \: 19.5303]$$

4.3 PID Controller

The electro hydraulic actuator might be capable of provide a precise positioning action, but its controller has significant influence on the output accuracy. The control algorithm starts by reading an error signal, which is obtained by subtracting the measured load position from the desired load position. The error signal then is processed by the control algorithm to develop a control signal. The goal of the control signal is to reduce the error signal to as close to zero as possible so that the desired and measured load positions are the same.

4.3.2 Structure of the controller

PID stands for Proportional, Integral, and Derivative. One form of controller widely used in industrial process due to their simple control structure, ease of design, and inexpensive cost, it is called a three terms, or PID controller. This controller has a transfer function:
A proportionality coefficient ($K_p$) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady state error. An integral coefficient ($K_I$) will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative coefficient ($K_D$) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

However, successful applications of the PID controller require the satisfactory tuning of parameters according to the dynamics of the process. In fact, most PID controllers are tuned on-site. The lengthy calculations for an initial guess of PID parameters can often be demanding if we know a few about the plant, especially when the system is unknown [14].

### 4.3.1 Rosaries Actuator Controller

The turbine unit no. 4, and by default the governor actuator has three modes of operation:

(i) No-load operation mode.

(ii) Isolated mode (connection to the internal power house network).

(iii) Interconnected mode, at which expose to fast transients.
To achieve acceptable comparison conditions, no any selection for PID parameters is done neither tuning, just straightforward simulation for the built in the loop PID controller, with operating (Kp), (Ki) and (Kd) coefficients. The functional block diagram showing the (PID) Function as built in Rosaries Unit no.4 guide vanes controller is shown in figure 4.2.

In the figure 4.2 (a) the (PIR3), stands for proportional integral ratios, is the (PI) portion of the control algorithm, with the following transfer function:

\[
F(s) = KP \frac{1 + T_n s}{T_n s} \tag{4.25}
\]

Where Tn is the reset time, and Kp is the controller gain. The derivative part as shown in figure 4.2 (b), is a function named (PTV) has the following transfer function;

\[
P(s) = K + T_v \frac{T_v s}{1 + T_1 s} \tag{4.26}
\]

The values of the (PID) controller parameters are listed in table 4.1 below;

Table 4.1: numerical values of the PID gains as built in real the plant

<table>
<thead>
<tr>
<th>Parameter</th>
<th>KP</th>
<th>Tn</th>
<th>Tv</th>
<th>T1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>2.39</td>
<td>10 sec</td>
<td>0.5 sec</td>
<td>4 sec</td>
</tr>
</tbody>
</table>
Figure 4.2: Function blocks showing the PID settings as built in Rosaries Unit no.4 Guide vane controller.
Chapter 5
Simulation and Results

5.1 System Identification
The real plant, so called governor actuator, consists of the following components:

5.1.1 Control Module
A micro controller based processing and control unit known as control module -type 83SR51-Figure 5.1- Mp on board is Motorola (MC68) family 12-bit. The board also contains many functional memory types listed in table 5.1 below;

<table>
<thead>
<tr>
<th>Memory contents</th>
<th>Storage type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating program</td>
<td>Flash-PROM</td>
</tr>
<tr>
<td>Standard Functions Library</td>
<td>Flash-PROM</td>
</tr>
<tr>
<td>User Program(structure, address, and simulation lists)</td>
<td>Flash-PROM</td>
</tr>
<tr>
<td>History values</td>
<td>RAM</td>
</tr>
<tr>
<td>Current module inputs and outputs signals.</td>
<td>RAM(shared memory)</td>
</tr>
</tbody>
</table>

The operating program allows onboard microprocessor to perform elementary operating functions of the module, while the standard function prom contains routines library ready to be used for the implementation of the various control functions.
The module is ready for operation, if a valid program is loaded into the user program flash-PROM; the former contains information on:

- How the function blocks are interconnected.
- Which physical inputs and outputs are assigned to which inputs and outputs of the function blocks.
- The fixed values (constants) specified at the individual inputs of function blocks.
- Parameters assigned at individual inputs of function blocks.
- Which function blocks support the process interfaces.
- The signals allowed to be simulated.

The functional diagram of the 83SR50 controller is presented in Appendix C.

![Control module 83SR51](image)

**Figure 5.1: Control module 83SR51**
5.1.2 Potential Buffer Amplifier

The potential buffer amplifier is a power electronic card that serves as an interface between the microcontroller circuit and the electrohydraulic transducer; also, it provides isolation of the digital circuit from the relatively high power force motor circuit. The amplifier accepts a set-point current in the range of (4 to 20 mA) from the control module and puts out a voltage in the range of (+/- 10 VDC) to the electrohydraulic transducer.

5.1.3 Electro Hydraulic Transducer

The transducer supplied on VATECH HYDRO governor is designed to accept a D.C. voltage signal and convert it by means of a moving coil type force motor, into a pilot valve position proportional to the strength of the input signal and in a direction dictated by the signal polarity.

The transducer unit is mounted on the main valve housing and controls the position of the main valve by hydraulic means. Mechanical feedback from the main valve to the transducer pilot valve ensures that proportionality will be maintained between pilot valve position and main valve position.

5.1.3.1 Electrical Part

The moving coil, an aluminum body carrying two control coils, moves axially in the circular air gap of the permanent magnet. This magnet is fixed to the top part of the housing in which the adjusting device for the moving coil is installed. By means of two
opposed adjusting springs the moving coil is held in central position.

The electrical connection between the moving coil and the connector is made via a terminal board by four flexible leads with AMP connectors. The technical data for each of the moving coils are presented in table 5.2;

Table 5.2: Technical data for moving coil force motor

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>max. allowable current</td>
<td>450 mA</td>
</tr>
<tr>
<td>Required current for (+/-2.5 mm) displacement.</td>
<td>200mA</td>
</tr>
<tr>
<td>Control volt</td>
<td>+/- 10 VDC</td>
</tr>
<tr>
<td>Resistance at 20º C</td>
<td>40 (for both parallel 20) ohm</td>
</tr>
<tr>
<td>Control capacity</td>
<td>3.6 w</td>
</tr>
<tr>
<td>Inductance</td>
<td>3 mH</td>
</tr>
<tr>
<td>Superimposed dither signal</td>
<td>25 mA (4-40 Hz)</td>
</tr>
<tr>
<td>Distributing valve piston movement caused by</td>
<td>0.05 mm</td>
</tr>
<tr>
<td>dither peak to peak</td>
<td></td>
</tr>
<tr>
<td>Pilot operating pressure</td>
<td>Max 40 bar</td>
</tr>
<tr>
<td>Quantity of pilot control oil</td>
<td>Approx 4.1/min</td>
</tr>
</tbody>
</table>
5.1.3.2 Hydraulic Part

The hydraulic control mechanism, consisting of a pilot control valve, is positioned in an aluminum housing which is bolted to the manual control valve housing. The pilot control valve is positioned between moving coil and main control feedback and connected by means of a lever. The technical data for the pilot stage are included in table 5.2 above.

5.1.3.3 Operation principle

Electrical leads from the moving coil are brought out to a standard screw type connector to which the output leads from the electronic controller are connected. The moving coil is held in central position by mechanical spring forces. The voltage on the coil is approximately 0,0 VDC at steady state of the turbine output. In case of power deviation a force is produced by the magnetic field of the energized moving coil and the magnetic field of the permanent magnet which is proportional to the corresponding control voltage and proportional to the displacement of the restoring force of the spring.

The amount of the displacement is determined by the current of the control coil, the direction of the displacement depends on the polarity. The full displacement in either direction
corresponds with a control current of 300 mA on the pertaining coil.

The moving coil is connected to one end of floating lever 1737E see figure 5.2. The opposite end of the lever is controlled by the main valve position through feedback lever 1701E and 1743E and link 1739E. The pilot valve 1719E is located between the ends of lever 1737E. It will be noted that when the force motor coil assembly moves off center, say in downward direction, the pilot valve also moves down and pressure oil is routed to the bottom side of the main valves control servo (A) causing the main valve to move upward until the pilot valve is restored to the ports blocked position, stopping the main valve movement. This arrangement insures proportionality being maintained between the pilot valve offset and the main valve offset.

Figure 5.2: Electro Hydraulic Transducer
5.1.3.4 Main valve dither

In order to keep the pressure on both the open and the close sides of the gate servomotors very near the minimum differential required to cause gate movement, it is necessary to dither or oscillate the main valve a few hundredth mm either side of the ports blocked position.

Also to achieve this and to avoid a possible sticking of the pilot control piston an adjustable alternating current of approx. 25 mA, 40 Hz/4 Hz is added to the control current on the moving coil. Response of the main valve to different dither frequencies (up to 40Hz) with constant input magnitude of 1.4 volts (25 mA) is shown in figure 5.3

![Figure 5.3: Main valve dither (1.4 v; 0.1 to 40 Hz)](image-url)
5.1.4 Hydraulic Cylinder Friction

In the previous chapter it is assumed that the friction force acting on the hydraulic cylinder piston is viscous friction. Here the procedure of determining friction coefficient is described.

To estimate the friction acting on the system, a dry operation (no-load open action) was carried out for the guide vane and runner blade actuator. The cylinder position was recorded each 0.3s for the first 6 seconds, from which the velocity and acceleration are determined offline. The set point and cylinder piston response are regenerated in the mat lab environment using linear interpolation, and applied as inputs to the system the resulted load force is recorded using the simulink model. Figure 5.4 shows the friction force versus speed.

Note that friction is a highly non-linear parameter that is due to its correlation with many physical parameters and environmental conditions. When two sliding materials are lubricated, different sliding speeds cause different film thicknesses of the lubricant and therefore friction characteristics may change. Another factor affecting the friction is the hydraulic cylinder chamber pressures as it will affect the surface area of the sealing in contact with the hydraulic cylinder wall. I used the following relation to calculate the friction force offline using mat lab;

\[
F_{vf} = F_L - m \frac{d^2x_{out}}{dt^2}
\]

Since we only considered the viscous friction force the slope of the linear portion of friction force vs. cylinder position chart was calculated and it found to be 197 Ns/mm for the guide vanes servomotor.
5.2 Frequency Response

In this sub-section the frequency of a sinusoidal signal is varied over a certain range and the resulting system response is studied. The open loop and closed loop frequency responses of the system are obtained throughout an experimental procedure and compared with the modeled system response. The dominant closed loop poles are chosen to determine the bandwidth of the closed loop position control hydraulic system. The desired bandwidth is 5 Hz. The linear state feedback controller gains corresponding to the desired closed loop pole locations are determined by following the procedure explained in Section 4.3. The experimental data in the time domain is transformed into frequency domain by using MATLAB built in functions. To find the frequency response of the system Fast Fourier Transforms (FFT) of the input signal and the system output are taken to determine the amplitudes of the constituting harmonics and their frequencies. FFT’s are taken with MATLAB "fft" command. The m-file script written for this purposes is given in Appendix B.

The written m-file generates a sinusoidal signal with exponentially decaying amplitude and linearly decreasing frequency with time. In the open loop test in order to prevent the
saturation of the hydraulic actuator, that is, to prevent the piston rod to reach the end of the stroke at low frequencies, this type of signal is generated. For the closed loop tests, constant amplitude sinusoidal test signals are generated with linearly increasing frequencies. This signal is the same as the MATLAB Simulink Chirp signal refer to figure 5.5.

5.2.1 Open Loop Frequency Response of the EHT

Using swept sinusoidal input signal in frequency range of 0.1 Hz to 12.5 Hz the electro hydraulic transducer frequency response had been examined, the result is shown in figure (5.6), the -3dB the exciting frequency is 9.5 Hz, that is compatible with the transducer manufacturer data it’s obvious that EHT band width greatly wider than the desired system band width this sustains our assumption that the EHT is not the primary element in the loop that determine system dynamic behavior, in the other hand it can drive the system faster to catastrophic oscillations, if badly controlled (tuned). The stroke versus flow relation and a factory test report are attached to appendix c.

Figure 5.5: Electro Hydraulic Transducer response to a chirp signal.
Figure 5.6: Frequency response of the EHT.
The frequency response of the EHT presented in figure 5.6 is obtained using a chirp signal generated by the code given in appendix B.

5.2.2 Open loop Frequency Response of the Actuator

The open loop test for valve controlled actuator model is performed using the mat-lab built in function (sisotool), which returns the magnitude phase plots for the system. The plots are shown in figure 5.8(a) and (b) below; it’s clear from the figure that The system behaves like an integrator as expected earlier in section 4.2, and from the Bode diagram we note that the theoretical resonance frequency of the system is around 70.4 Hz, also at low frequency regions, both the non-linear model and the linearized one rapidly responses to the input signal and there is no need to wait for the system to catch steady state. Thus continuously changing the test signal frequency is not a problem for this frequency response tests.

Due to the free s term in the open loop transfer function between the valve spool position and hydraulic cylinder position, there occurs a 90 degrees phase shift at low frequency region. Figure 5.7 shows the hydraulic cylinder position response and illustrates why an exponentially decaying amplitude sinusoidal signal should be chosen as the test signal. By decreasing the amplitude and frequency with time saturation of the hydraulic cylinder is prevented. Two different open loop frequency response graphs are presented one for the linearized mathematical model (extension- case) of the valve controlled system. This is due to the inherent property of the single rod cylinders that different extending and retracting speed exist.

The open loop transfer function for the linearized system is calculated using the equations and numerical values presented in chap.3, and found to be:
Figure 5.7: System response to exponentially decaying sinusoidal signal.
Figure 5.8: Open loop frequency response (a) nonlinear EHA. (b) linearized Actuator
5.2.3 Close loop frequency response of the EHA

To achieve an accurate comparison between the two controllers, the same test signal is applied. The linear state feedback gains corresponding to desired closed loop pole locations are determined by following the procedure explained in Section 4.2. Throughout all the frequency response tests, the supply pressure is kept at 35bar (3.5 Mpa). The test signal properties, the desired closed loop poles and the corresponding state feedback gains are listed in Table 5.3

Table 5.3: Frequency Response simulation Data

<table>
<thead>
<tr>
<th>Reference Chirp Signal</th>
<th>Magnitude</th>
<th>Start Frequency</th>
<th>Stop Frequency</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0.01 Hz</td>
<td>100.01 Hz</td>
<td></td>
<td>80 s</td>
</tr>
<tr>
<td>Desired Closed Loop Poles</td>
<td>[-2*2π,700,800]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>State Feedback Gains</td>
<td>Extension</td>
<td>[1.2028 0.0546 18.6307]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Retraction</td>
<td>[1.2967 0.0574 19.5303]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linearized at</td>
<td>Supply Pressure</td>
<td>3.5Mpa</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Spool position</td>
<td>1.4mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.9 shows the response of the non-linear model. The yellow signal is the reference position signal 10%, while the violet one is model response dominated by the state feedback gain controller and the blue one is the system response under a PID controller with the same gains as the real plant.

Also, at higher frequencies, it is seen that the system and the non-linear model responses seem to track not an exact sinusoidal profile, but rather a ramp like profile. This the result of switching type controller strategy with the gains calculated according to the linearized system equations, if the same controller is to be applied on the linearized model, it will be seen that the response profile is exactly sinusoidal.
Figure 5.9: Closed loop system response of the non-linear EHA.

(a) Figure 5.10: Load pressure at (a) lower (b) higher, frequency regions.

(b)
Figure 5.10.a shows the load pressure response of the system, during the sine sweep test in a detailed view at lower frequency region, it is seen that the load pressure tracks a square wave like profile.

This is due to the friction of the cylinder. However, this non-linear load pressure characteristic is not reflected to the generated input signal. In the third plot of the Figure 5.10.b the detailed view of the load pressure responses at higher excitation frequency is shown, it is seen that the effects of the friction on the load pressure is reduced and the real load pressure is consistent with the model response.

In Figure 5.11: the frequency response of the non-linear model response under control of both state feedback gain controller, and the PID controller tuned with the same gains existing in the real unit no.4 guide vane actuator in Rosaries Hydro Power plant the plot description is detailed in the figure legend, the magnitude and phase angle are calculated using the m-code attached to appendix B.
Figure 5.11: Frequency Response of non-linear model under control of state gain controller and PID controller and the linearized model.
Note that because the desired closed loop pole locations for extension and retraction are the same, the dynamic response of the closed loop system for extension and retraction are identical, therefore unlike from the open loop frequency response graph, there exists only one frequency response curve defining the closed loop system characteristics.

In Figure 5.11 it is seen that, the magnitude plot of the linearized model reflects the desired closed loop system behavior. The magnitude of the closed loop frequency response is \(-3\text{ dB}\) at 2 Hz excitation frequency, indicating the bandwidth of the system. This is an expected result, because the desired closed loop poles are located at \([-2\times2\pi, -700, -800]\).

Because the last two poles are far away from the imaginary axis with respect to the first pole, the pole located at \(-2\times2\pi\) rad/s dominates the system characteristics, and resulting in a 2 Hz bandwidth of the closed loop system. However, the non-linear model response is not consistent with the linearized model response at higher frequencies. This is the result of linearization, with the increasing excitation frequency the operating points where the linearization is performed changes.

For example, the valve gains are linearized at steady state operating pressures both for extension and retraction, the steady state chamber pressure values are constant and do not change with the spool position, but the spool direction. However, with the increased excitation frequency when the valve spool changes direction the time passed in transient period dominates the total excitation frequency period, resulting in a different system behavior than the linearized one.
5.3 Step Response of the system

The test signal properties, the desired closed loop poles and the corresponding state feedback gains are listed in Table 5.4. The corresponding linear state feedback gains of the valve controlled system are determined through the linearized system equations defined in Section 4.2. Because the single rod cylinder has inherently different characteristics for extension and retraction, two set of linear state feedback gains are calculated.

Table 5.4: Step Response Simulation Data

<table>
<thead>
<tr>
<th>Reference Step Signal</th>
<th>Magnitude</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0.2 Hz</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Desired Closed Loop Poles</th>
<th>[-0.92*2π,700,800]</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Feedback Gains</td>
<td></td>
</tr>
<tr>
<td>Extension</td>
<td>[0.4  0.0546  18.6307 ]</td>
</tr>
<tr>
<td>Retraction</td>
<td>[1.2967  0.0574  19.5303 ]</td>
</tr>
</tbody>
</table>

| Linearized at             |                   |
| Supply Pressure           | 3.5Mpa             |
| Spool position            | 1.4mm              |

Figure 5.12 shows the step response of the three systems with step magnitude of 100%, and a frequency of 0.05 given the closed loop desired poles has been shifted such that system bandwidth is just 1Hz.

Note that the desired closed loop pole that dominates the system behavior is located at -0.9*2π rad/s. Because the other two poles (-700.rad/s,-800.rad/s) are located very far to the left of the desired closed loop pole, their effects on the response can be assumed to be negligible, so that the closed loop position control system can be thought as a first order system with the following transfer function.

\[
\frac{X_{out}(s)}{X_{reg}(s)} = \frac{1}{Ts + 1}
\]
The time constant here is;

\[ T = \frac{1}{0.9 \cdot 2 \cdot \pi} = 0.1769 \text{ sec} \]

Time constant \( T \) is an important parameter of first order systems, because at time \( t=T \), the response of the system reaches 63.2\% of its total change. This can be verified from the linearized system response, at time 0.2 sec the system response is 63.12\%.

Also it’s the steady state error for the PID controlled non-linear model 0.2\%, while for the it is zero for the state feedback controlled and the linearized models this is expected since the open loop response was found to be of type 1, with free \((s)\) in the denominator.

The non-linear model behavior under control of PID and state feedback gains is different of the linearized model this is mainly due to the opening limits in the pilot stage valve this can easily be checked by removing the saturation limits, when this done the none linear system response reached a 63.2\% in 1.017 sec instead of 3.3 sec with the limiters.
Figure 5.12: Step response of the non-linear and linearized models
5.4 Comparison

Due to the inherent property of the single rod hydraulic actuator with unequal cylinder areas, the flow rate entering the cap end side chamber is not equal to the flow rate exiting from the rod end side. The asymmetric flow rate of the hydraulic actuator results in such a non-linearity that different steady state chamber pressures exists according to the valve spool position; causing different valve spool position gains and different extension and retraction speeds [15].

The different dynamic characteristics of the single rod asymmetric cylinder for extension and retraction bring about the necessity to use different controller gains for extension and retraction. However switching the controller gains according to spool position causes some-what oscillatory-rugged behavior on the hydraulic actuator position response at switching times. Of course, this unwanted property can be eliminated by modifying the control strategy, but this brings another complexity.

The State feedback gains technique exhibit robust tracing behavior specially at low frequency regions, but unfortunately it is suffering gain switching problems and more expensive than a PID.

The switched gains controller is more sensitive to disturbances in feedback signals which make tuning of the controller very hard.

Compared to the commissioning field test data, shown in Figure 5.13, the non-linear model dominated by the PID controller is the best represent the actual system with a settling time of 14.17 sec. the slight differences near steady state in the step response test are due approximation errors and uncertainties in friction estimation using a real plant response data.
Figure 5.13: Settling time of (a) non-linear model (b) real plant response.
Chapter 6 Conclusions and Future work

6.1 Conclusions

The accomplished works in this thesis are:

- Background about the system subject of thesis and energy efficiency budget.
- Modeling of the Electro Hydraulic Actuator in MATLAB Simulink environment.
- Derivation of linear and linearized reduced order differential equations defining the system dynamics;
- Linear state feedback controller design by using the reduced order linear and linearized system equations;
- System identification and finding the un-measurable quantities off line.
- Carrying out a performance tests and results investigated.
- Comparison of the real plant response with the non-linear model response, and the state feedback controller performance with the PID one.
At the beginning of the study, brief background about the system subject of the thesis is presented, all plant elements illustrated quite enough. In the sections at the beginning of the study, detailed mathematical model of the EHA is developed. For simplification, the dynamics of the pilot stage electro hydraulic transducer (EHT) assumed to be ideal; this assumption relatively validated since it’s bandwidth proved to be wider than the actuator.

A non-linear model of the (EHA) consisting of the pilot stage, the hydraulic actuator, and the load dynamics is developed in the MATLAB Simulink environment. The system also modeled analytically to fully understand its dynamics. The cylinder dynamics accompanied with the load dynamics results in a 3rd order differential equation between the actuator input and the hydraulic cylinder velocity response. However, when the relation between the dynamic change of hydraulic cylinder chamber pressures is investigated, it is seen that dynamic pressure changes in the hydraulic cylinder chambers become linearly dependent above and below some prescribed cut off frequencies. Thus, assuming linearly dependent chamber pressure response, the order of the dynamic equations defining the system dynamics is reduced, resulting in a 2nd order transfer function between the actuator input and the hydraulic cylinder velocity. By this way, the parameters affecting the system dynamics of the system are explained clearly. The valve flow characteristic equation is linearized at steady state chamber pressures for extension and retraction at a given spool position to derive a transfer function for the system from the block diagram representations drawn for the open loop response of the actuator in chap. 3. For the position control of the single rod hydraulic actuator, it is decided compare the performance of two controllers: one a PID tuned as existing in
unit no.4 guide vane actuator and a linear state feedback control (switched gain type). Because of the equal cylinder annulus faces areas two different system dynamics for extension and for retraction are resulted. For this reason, two different state feedback gain sets are determined for extension and for retraction. In the applied control algorithm the state feedback gains are switched according to the valve spool position command polarity.

Lastly, a performance test carried out non-linear system compared with the real plant, and for the two controller in which the non-linear model verified to be best proximity to real system in combination with a PID controller.

One of valuable benefits of the research is setting our focus on the ability to minimize energy losses in the existing system. It is shown that the maximum efficiency of a conventional valve controlled circuit is 38.5%, and noted that this is valid for only at an instant of time when the maximum power requirement is equal to the maximum power input of the valve, if the total duty cycle of the load is considered, the efficiency of the hydraulic circuit will be lower than this figure. It should be remembered that the fluid power energy lost on the servo valve and the relief valve transforms into heat energy, warming up the hydraulic oil. Hydraulic oil characteristics change with the increasing oil temperature thus necessitates for cooling of the hydraulic oil arises in the valve controlled system. This should be accounted for another additional energy loss.
6.2 Future Work

Building a physical actuator prototype depending on the implemented non-linear model, interface it to a computerized engineering system, because this will open the way to get deep practice and understanding in electro hydraulic actuation systems, for the plant engineers and the universities students who annually visits the power station.

More optimization for the model, especially in the estimation of the friction characteristics, both for the guide and runner blades actuators, after that the linearized models can be used confidently to study the Kaplan turbines dynamics.

Since, most of the energy loss is due to throttling losses; this means most of the power losses is not due to regulate the flow rate through the hydraulic actuator but to supply a constant pressure for the servo valve intake. Long time in the pump operation cycle is for circulation of oil waiting for loading conditions, so great amount of the flow delivered by the pump to the system passes through the relief valve to the oil tank, accompanying with a pressure drop equivalent to the valve supply pressure. One way to reduce the power loss on the relief valve is to decrease the speed using (VSD) variable speed drive. However, this may result in fluctuations in the supply pressure during actuation transients, so a deep study is needed to improve energy efficiency without affecting system performance.

Last, the friction characteristics are not characterized perfectly, so additional work should be done to include it is effect to the model for more realistic system behavior.
REFERENCES


[11] High Bandwidth Electrohydraulic Pilot Valve Actuator; Sabri Cetinkunt Professor; University of Illinois at Chicago

[12] Mechanical Science; Volume 1 of 2; DOE- HDBK JANUARY 1993


APPENDICES

APPENDIX A:

Transfer Function Derivation for the Actuator

To be uniform and perceptible all the dynamic equations that define the pump controlled system are repeated below. Because the procedure is the same, the transfer function is derived only for the extension of the hydraulic actuator. The linearized valve flow characteristic equations, and The flow continuity equations of the hydraulic cylinder are:

\[ q_2 = k_v u \sqrt{p_s - p_A} = k x_2 \text{\_\_ ext } u - k p_2 \text{\_\_ ext } \cdot p A \]
\[ q_4 = k_v u \sqrt{p B} = k x_4 \text{\_\_ ext } u + k p_2 \text{\_\_ ext } \cdot p B \]  
\[ \text{(1)} \]

\[ q_A = A_A \cdot x' + \frac{V_A}{E} \cdot \frac{dp_A}{dt} \]
\[ \text{(2)} \]

\[ q_B = A_B \cdot x' - \frac{V_B}{E} \cdot \frac{dp_B}{dt} \]
\[ \text{(3)} \]

The load pressure is:

\[ P_L = \gamma p_A - p_B \]
\[ \text{(4)} \]

The structural equation of the load:

\[ p_L \cdot A_B = m_x \cdot + b_x \cdot \]
\[ \text{(5)} \]

The continuity equations necessitate s that:

\[ q_2 = q_A \]
\[ q_4 = q_B \]
\[ \text{(6)} \]

Substituting equ.(1) and equ.(3) int o equ.(7) and equ.(2) and equ.(4) int o equ.(8) results:

\[ k x_4 \text{\_\_ ext } \cdot u - k p_2 \text{\_\_ ext } p A = A_A \cdot x' + \frac{V_A}{E} \cdot \frac{dp_L}{dt} \]
\[ \text{(9)} \]

\[ k x_4 \text{\_\_ ext } \cdot u + k p_4 \text{\_\_ ext } p_B = A_B \cdot x' - \frac{V_B}{E} \cdot \frac{dp_B}{dt} \]
\[ \text{(10)} \]
making the following substitutions

\[ A_A = \gamma A_B \]  
\[ k_{x2 \_ext} = \gamma k_{x4 \_ext} \]  
\[ k_{p2 \_ext} = \gamma k_{x2 \_ext} \]  
\[ V_A = \alpha V_B \]

then after arrangement:

\[ \gamma k_{x4 \_ext} \cdot u - \gamma A_B \cdot x - k_{p2 \_ext} \cdot P_A = \frac{\alpha V_B}{E} \cdot \frac{dp_A}{dt} \]  
\[ k_{x4 \_ext} \cdot u - A_B \cdot x + \gamma k_{p2 \_ext} P_B = -\frac{V_B}{E} \cdot \frac{dp_B}{dt} \]

Taking the Laplace transform, and rearranging:

\[ \gamma k_{x4 \_ext} \cdot u(s) - \gamma A_B \cdot sX(s) = P_A(s) \]  
\[ \gamma k_{p2 \_ext} + \frac{\alpha V_B}{E} \cdot k_{p2 \_ext} + \frac{\alpha V_B}{E} \cdot sX(s) = P_A(s) \]  
\[ k_{x4 \_ext} \cdot u(s) = -\frac{A_B}{k_{p2 \_ext} s} + \frac{V_B}{E} \cdot sX(s) = P_B(s) \]  
\[ \gamma k_{p2 \_ext} + \frac{V_B}{E} \cdot s \]  
\[ \gamma k_{p2 \_ext} + \frac{V_B}{E} \cdot s \]
Multiplying the equation (17) by the area ratio $\gamma$ and summing with equation (18) gives

$$\gamma^2 k_{x4\_ext} (\gamma k_{p2\_ext} + \frac{V_B}{E} s) + k_{x4\_ext} (k_{p2\_ext} + \frac{aV_B}{E}s)$$

$$\frac{(k_{p2\_ext} + \frac{aV_B}{E}s)(\gamma k_{p2\_ext} + \frac{V_B}{E}s)}{(k_{p2\_ext} + \frac{aV_B}{E}s)(\gamma k_{p2\_ext} + \frac{V_B}{E}s)} U(s)$$

$$\gamma^2 k_{x4\_ext} (\gamma k_{p2\_ext} + \frac{V_B}{E} s) + A_B (k_{p2\_ext} + \frac{aV_B}{E}s)$$

$$\frac{(k_{p2\_ext} + \frac{aV_B}{E}s)(\gamma k_{p2\_ext} + \frac{V_B}{E}s)}{(k_{p2\_ext} + \frac{aV_B}{E}s)(\gamma k_{p2\_ext} + \frac{V_B}{E}s)} sX(s) = \gamma P_A(s) - P_B(s)$$

Inserting equation (5) and equation (6) into equation (19) and rearranging, we obtain:

$$\frac{(\gamma^3 + 1)k_{p2\_ext} + (\gamma^2 + \alpha)\frac{V_B}{E}s}{\alpha\frac{V_B}{E^2}s^2 + k_{p2\_ext} \frac{V_B}{E} (\gamma\alpha + 1)s + \gamma k_{p2\_ext}} k_{x4\_ext} U(s)$$

$$\frac{\frac{aV_B}{E^2}s^2 + k_{p2\_ext} \frac{V_B}{E} (\gamma\alpha + 1)s + \gamma k_{p2\_ext}^2}{A_B s} X(s) = \frac{ms^2 + bs}{A_B} X(s)$$

Arranging equation (20) again, the transfer function between the valve spool position and the hydraulic actuator velocity is given as

$$\frac{V(s)}{Xin(s)} = \frac{a_1 s + a_2}{b_1 s^3 + b_2 s^2 + b_3 s + b_4}$$

$$a_1 = k_{x4\_ext} A_B (\gamma^2 + \alpha) \frac{V_B}{E}$$

$$a_2 = k_{x4\_ext} A_B (\gamma^2 + \alpha) \frac{V_B}{E}$$

$$b_1 = \frac{m\alpha V_B^2}{E^2}$$

$$b_2 = mk_{p2\_ext} \frac{V_B}{E} (\gamma\alpha + 1) + \frac{b\alpha V_B^2}{E^2}$$

$$b_3 = m \gamma k_{p2\_ext}^2 + bk_{p2\_ext} \frac{V_B}{E} (\gamma\alpha + 1) + (\gamma^2 + 1) \frac{V_B}{E} A_B^2$$

$$b_4 = b\gamma k_{p2\_ext}^2 + (\gamma^3 + 1)k_{p2\_ext} A_B^2$$
Reduced Order Transfer Function Derivation for the Actuator in Extension case is explained below.

Multiplying Eq.(15) with the area ratio (gama) and multiplying Eq.(16) with the volume ratio (alfa),

\[ \gamma^2 k_{x4, ext} \cdot u - \gamma^2 A_B \cdot x' - \gamma k_{p2, ext} \cdot p_A = \frac{\alpha V_B}{E} \cdot \gamma \frac{dp_A}{dt} \]  \tag{22}

\[ \alpha k_{x4, ext} \cdot u - \alpha A_B \cdot x' + \gamma \alpha k_{p2, ext} p_B = - \frac{\alpha V_B}{E} \cdot \frac{dp_B}{dt} \]  \tag{23}

and assuming the resulting expression gives the rate of load pressure change \( p_L \) as:

\[ (\gamma^2 + \alpha) k_{x4, ext} \cdot u - (\gamma^2 + \alpha) A_B \cdot x' - k_{p2, ext} \cdot p_A + \gamma \alpha k_{p2, ext} p_B = \frac{\alpha V_B}{E} \cdot p_L \]  \tag{24}

Assuming that the dynamic chamber pressure changes \( p_A \) and \( p_B \) are linearly dependent and defined by:

\[ p_A = - \varphi p_B \]  \tag{25}

and through Eq.(5) and Eq.(25) writing the dynamic chamber pressure changes \( p_A \) and \( p_B \) in terms of load pressure \( p_L \) as:

\[ p_A = - \frac{\varphi p_L}{\gamma \varphi + 1} \]  \tag{26}

\[ p_B = - \frac{p_L}{\gamma \varphi + 1} \]  \tag{27}

and substituting Eq.(26) and Eq.(27) into the Eq.(24) give

\[ (\gamma^2 + \alpha) k_{x4, ext} \cdot u - (\gamma^2 + \alpha) A_B \cdot x' - \frac{\varphi + \alpha}{\gamma \varphi + 1} \cdot \gamma k_{p2, ext} \cdot p_L = \frac{\alpha V_B}{E} \cdot p_L \]  \tag{28}

Rearranging and taking the Laplace transform of above expression, assuming zero initial condition give:

\[ (\gamma^2 + \alpha) k_{x4, ext} \cdot U(s) - (\gamma^2 + \alpha) A_B \cdot X(s) = \left( \frac{\alpha V_B}{E} \cdot s + \frac{\varphi + \alpha}{\gamma \varphi + 1} \cdot \gamma k_{p2, ext} \right) P_L(s) \]  \tag{29}
Taking the Laplace transform of Eq.(6) and inserting into Eq. (29) give:

\[(\gamma^2 + \alpha)k_{x4\_ext}A_B U(s) - (\gamma^2 + \alpha)A_B^2 sX(s) = \left(\frac{aB}{E} + \frac{\varphi + \alpha}{\gamma \varphi + 1} \gamma k_{p2\_ext}\right)(ms + b)sX(s)\]

Simplifying the above expression, the transfer function between the valve spool position and hydraulic actuator piston speed is obtained as:

\[
\frac{V(s)}{U(s)} = \frac{(\gamma^2 + \alpha)k_{x4\_ext}A_B}{m\alpha B s^2 + (m\varphi + \alpha)\gamma k_{p2\_ext} + \frac{b\alpha B}{E}s + b\varphi + \alpha \gamma k_{p2\_ext} + (\gamma^2 + \alpha)A_B^2}
\]
APPENDIX B:

M-code to calculate input output magnitude and phase angle:[1]

```matlab
Pos(:,1)= xout;
Sp(:,1)= RefPos; % Reference Position

fs=1000; % Sampling Rate [Hz]
tstart=0.1; % Start Time [s]
tend=80.1; % End Time [s]
FreqMin=0.01; % Minimum Frequency [Hz]
FreqMax=100.01; % Maximum Frequency [Hz]
Freq_Inc=0.78125; % Frequency Increment [Hz]
for i=1:1
    out(:,i)= Pos(:,i);
    % in(:,i)=input(tstart*fs:tend*fs,i);
    % Remove the 'linear' trend of the output
    out(:,i)=detrend(out(:,i));
    % Calculate the FFT of the input and the Output
    % in_fft(:,i)=fft(in(:,i));
    out_fft(:,i)=fft(out(:,i));
end
in(:,1)=Sp(:,1);
in_fft(:,1)=fft(in(:,1));

% Take the Avarage FFT
for i=1:length(out_fft)
    out_fft_mean(i,1)=mean(out_fft(i,:));
% in_fft_mean(i)=mean(in_fft(i,:));
end
t=0:1/fs:(tend-tstart);
% Frequency Array
FreqArray=0:fs/(length(in_fft)-1):fs;
% Bode Plot

Mag=20*log10(abs(out_fft_mean)./abs(in_fft));
```
PhsAngle=(-angle(in_fft)+angle(out_fft_mean))*180/pi;
f=FreqMin;
j=1;
for i=1:(length(Mag)-1)
    if PhsAngle(i+1,1)-PhsAngle(i,1)>200
        PhsAngle(i+1,1)=PhsAngle(i+1,1)-360;
    end
    if PhsAngle(i+1,1)-PhsAngle(i,1)<-200
        PhsAngle(i+1,1)=PhsAngle(i+1,1)+360;
    end
    if FreqArray(i)<FreqMax
        if FreqArray(i)>f
            x(j)=FreqArray(i-1);
            y_mag(j)=Mag(i-1);
            y_phase(j)=PhsAngle(i-1);
            f=f+Freq_Inc;
            j=j+1;
            plot(x,y_mag,y_phase);
        end
    end
end

Mat-lab code for sinusoidal chirp signal for open loop test:

clc
clear all

fs=10000;
t=(1:10000)/fs;
for f=1:13
data(f)=cos(2*pi*log(f)*t(f));
end
APPENDIX-C

- 83SR50 Control module function diagram:
- Potential Buffer Amplifier-VE 14 layout:
- Electro Hydraulic Transducer:

Fig 10. Flow curve of TR-h 7/..F related on pressure drop of 10 bars.
Electro Hydraulic Transducer Factory Test report:

### Table: Workshop Test and At Site

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<thead>
<tr>
<th>Workshop Test</th>
<th>At Site</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Signal [V]</strong></td>
<td><strong>Measured Input Current [mA]</strong></td>
</tr>
<tr>
<td>-10,00 V</td>
<td>-280,0</td>
</tr>
<tr>
<td>-8,00 V</td>
<td>-230,0</td>
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<td>-6,00 V</td>
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### Servomotor Stroke and Volume

<table>
<thead>
<tr>
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<th>Servomotor Volume</th>
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<tbody>
<tr>
<td>mm</td>
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</table>

### Notes: / Примечания

Recorded by / Записано: Regulator VA TECH HYDRO / LH-PT

Approved by / Подписано: VA TECH HYDRO