ABBRIVIATIONS

FP_o:fractional programming problem; other form ofDEA, measures efficiency ,mean that the ratio of "virtual output" vs. "Virtual input" should not exceed 1 for each

LPo:Linear programming problem; other form ofDEA,measures efficiency of for each DMU, mean that there are linear relation between variables.

DLPo:dual Linear programming problem ;has a feasible solution □ = 1 , $\square_o$ = 1, □ = 1 ($j \neq 0$) .looking for input that guarantees at least the output level Y_o of DMUo in all components while reducing the input vector x_o proportionally to a value as small as possible .

TFP:Total Factor Productivity indexapproach; itis the chain, which measures a change in efficiency relative to a base year.

CHAPTER THREE

METHODOLOGY OF THE STUDY:

3.1 Introduction:

This chapter covers some concepts related tothemethodology of the study ;(DEA) and Total Factor of productivity (TFP) Index Approach. This methodologyis based on operation research concepts and theories. The study builds on some operation research principles, which search for optimal solutions among different available solutions in order to reach the efficiency level of productive system using inputs and outputs data. The research is searching for the optimal relationship between production inputs and outputs in the Sudanese mechanized rain fed and Agricultural schemes in the two states.

To evaluate efficiency, and examine total factors productivity change in Sudanese agricultural products ,the study employed both the Data Envelopment Analysis (DEA) models and Malmquist(Total Productivity Factors) index .In attempting (DEA) approach the study uses both the two DEA models :CCR (Charnes, Cooper , and Rhodes , 1978) , and the BCC (Banker, Charnes, and Cooper, 1984). These DEA two models are employed to evaluate and measure efficiency. In addition, Malmquist (TFP) index is used to measure efficiency changes in the Sudanese agricultural productivity over the period (2000- 2010).All these models are used under the two assumptions: constant return to scale (CRS), and variable return to scale (VRS). The following are brief explanation of each approach.

3.2. Basic DEA models:

DEA can be seen as extension of fractional programming analysis since it enables us to consider the use of multiple inputs which produce multiple outputs. Under the alternative assumption of constant return to scale (CRS) and variable return to scale (VRS) it is possible to decompose the technical efficiency score into the component of pure efficiency and scale efficiency. Accordingly, it is possible to determine if the individual DMUs is experiencing increasing constant or decreasing return to scale, a separate production frontier can be estimated for each year of this study.

InDEAthe organization under study is called DMU (Decision Making Unit) the definition of DMU is rather lose to allow flexibility in its use over the wide range of possible application. Generally a DMU is regarded as the entity responsible for converting inputs into outputs' and whose performance is to be evaluated.

DEA is a non-parametric technique for evaluating the technical efficiencies of a collection of "Decision making units (DMUs)" (e.g. bank branches, Crown Health Enterprises) which use common inputs to generate common outputs). A DMU is said to be 100% efficient if:

(a)None of the outputs can be increased without either:

(i) Increasing one or more inputs, or

(ii) Decreasing some of the other outputs.

(b) None of the inputs can be decreased without either

(i) Decreasing some of its outputs.

(ii) Increasing some of its other inputs.

To employ DEA in efficiency studies, annual report of inputs and outputs must be specified , the outcome is to produce convex production frontier for output orientation , while the input orientation produce concave production frontier .DEA generate within- sample efficiency score between 1 and 0 , with 1 being most efficient .

DEA approach as a mathematical optimization technique looks for determining the relative efficiency of production frontier, based on empirical data on chosen inputs and output of a number of entities called decision making units (Boris, Igor, 2001) . From set available data DEA identify references point(relatively efficient DMUs) that defined the efficient frontier (as the best practice production technology) and evaluate the efficiency of other , interior point (relatively inefficient DMUs) that are below this frontier . Ensuring that the efficiencies of other units do not exceed 100%, besides, determining relative efficiency measures for each DMU. DEA also identify efficient peer DMUs for each inefficient DMU and quantifies the required increase in outputs or decrease in inputs required to transform an inefficient DMU into an efficient DMU. It can help in answering the question "How can they improve efficiency in agricultural production?

Now, there is an extensive literature discussing both theory and applications of DEA. While applications have been reported in the private sector (e.g. in retailing, banking, hotels and the airline industry), most of these applications have occurred in the public sector for instance, In New Zealand the use of DEA is promoted the health sector. Also DEA is involved in schemes s in retailing, audit risk evaluation, and in agriculture in which it has been used to measure performance and quantify inefficiencies.

Several alternative DEA models have employed in banks efficiency literature .The DEA models differ according to the difference in the shape of the efficient frontier. Therefore, this approach has several advantages as follow:

First: is suitable for assessing the efficiency of public sector nonprofit organization where multi inputs produce using multi outputs, when there are no needs to inputs and outputs prices data.

Second: it provide quantitative information on extent of inefficiency and subsequently on the largest requireto become efficient.

Third: it identifies best practice rather than average performance which may serve as benchmark for inefficient schemes. DEA has become increasingly popular in measuring efficiencies in different private and public institutions.

The CCR and BCC models differ as the former evaluates scales as well as technical inefficiencies. For a DMU to be considered as CCR efficient, it should be both scale and pure technically efficient. For a DMU to be BCC efficient, it only needs to be pure technically efficient. As a result, the ratio of CCR efficiency score over the BCC score gives the scale efficiency index. The main objective of a DEA study is to put the inefficient DMUs onto the most efficient frontiers of the DMUs in the sample, under the assumption of constant return to scale (CRS) and variable return to scale (VRS). There are to direction; input orientation approach that aims at reducing the input amount by as much as possible at a given level of output, and the output orientation approach that maximizes output level at a given input level. Below is a brief discussion of the main concept of DEA and its model behind the Malmquist (total factors productivity) index concept which measures change in efficiency and total productivity growth change.

According to DEA approach, suppose there are a number of DMUs which have certain degree of managerial freedom in decision making, so as to evaluate the efficiency of this units we need the following:

1- Availability of numerical data for each inputs and outputs,assumingthat the data to be positive for all DMUs.

2- The items (inputs, outputs and choice of DMUs) should reflect the interest of the analyst or the manager in the components that will enter into the relative efficiency evaluations of the DMUs.

3 - In principle, smaller inputs amounts are preferable and larger outputs amounts are preferable, so the efficiency scores should reflect these principles.

4- The measurement units of the different inputs and outputs need not be congruent. Some may involve number of persons, or areas of floor space,money expended, etc.

3.2.1. The basic CCR model:

This model of DEA whichdeveloped by Charnels, Cooper and Rhodes (1978),is a non-parametric technique used for evaluating the efficiencies of a collection of "Decision Making units ,this models introduces a measure of efficiency for each DMU that is obtained as maximum of a ratio of weight outputs to weight inputs. the weight for the ratio determined by restriction that similar ratio for every DMU have to be less than or equal to 1 , thus reducing multiple inputs "virtual "and single "virtual" out puts without requiring changed weight .

The efficiency measure is then functioning weight of the "virtual" inputs – outputs combination.

Given the data; it is possible to measure the efficiency for each DMU once, and hence need no optimization , one for each DMU_j to be evaluated .let the DMU_j to be evaluated on any trial be designed as DMU_0 where 0 ranges over (1, 2, …,n) .

So, according to the above, measuring efficiency for a number of agricultural schemes s , we need to solve the following fractional programming problem to obtain value for input " weight" (V_j)

(j=1,…,m) and output "weight" " (u_r) (r =1,…,s) as the variables .

$$\frac{u_1 Y_{10}+u_2 Y_{20}+\dots+u_s Y_{so}}{V_1 x_{10}+V_2 x_{20}+\dots+V_m x_{mo}}$$ (1. 1)Max □ = (FP_O)

Subject to = $\frac{u_1 Y_{1j}+\dots+u_s y_{sj}}{s_1 x_{1j}+\dots+V_m x_{mj}}$ ≤ 1 (j=1,…,n) (1.2)

V_1 , V_2 , …, V_m ≥ 0 (1.3)

U_1 , U_2 ,…, U_s ≥ 0 (1.4)

The constraint mean that the ratio of "virtual output" vs. "Virtual input" should not exceed 1 for each DMU. The objective is to obtain weights (V_r and (U_r) that maximize the ratio of DMUo, the DMU being evaluated. by virtue of the constraints, the optimum objective value is at most 1 .mathematically the non-negativity constraint (1.3) is not sufficient for the fractional term (1, 2) to have a definite value. The weight of all input and output has positive value.

i. **From fractional to a linear program:**

Further, we can shift from fractional relation to the linearrelation by the following linear program (LPo):

(LPo) MAX = $\mu_1 Y_{1o}$ + …+ µ s Y s o (1.5)

Subject to: $V_1 x_{1o}$ +…+ $V_m x_{mo}$ = 1 (1.6)

$\mu_1 Y_{1j}$ + $\mu_s Y_{sj}$ ≤ $v_1 x_{1j}$ +… $v_m x_{mj}$ (1. 7)

($j=$ 1,…,n)

v_1 , v 2 ,…, v m ≥ 0 (1.8)

μ_1 , μ_2 ,…, μ_s ≥ 0 (1 .9)

Definition (1.1) (CCR-inefficiency):

1- DMUo is CCR – inefficient if □* = 1 and there exist at least one optimum ($v^{¿}$, $u^{¿}$) with v*> 0 and $u^{¿}$ > 0.

2- Otherwise, DMUo is CCR- inefficient.

Thus , CCR efficiency means that either (i) □* < 1 ,or (ii) □* = 1and at least one element of ($v^{¿}$, $u^{¿}$) is zero for every optimum solution of (LPo).

In case where DMUo□* < 1 (CCR – inefficient) , there must be at least one constraint (or DMU) for which the weight ($v^{¿}$, $u^{¿}$) produces equality between the left and the right hand side since, otherwise □* could be enlarged. Let the set of such j $\in\{1,\ldots,n\}$ be

$$E_O^1=\{j:\sum_{r=1}^{8} u_r^{¿} Y r j=\sum_{i=1}^{m} v_r^{¿} x r i j\}. \quad (1\ .8)$$

The best set E_O of E_O^1 composed of CCR-efficient DMUs, is called the reference set or the peer group to the DMU_O .

It is the existence of this collection of efficient DMUs that the DMUo to be inefficient. The set spanned by E_o is called the efficient frontier of DMUo.

ii. Meaning of optimum weight

The ($v^{¿}$, $u^{¿}$) obtained as an optimum solution for (LP_o) resulting in asset optimum weight for the DMU_o , the ratio scale is evaluated by :

$$\square^* = \frac{\sum_{r=1}^{8} u_r^{¿} Yro.}{\sum_{i=1}^{m} v_i^{¿} xio.} \quad (1.9)$$

From (2,8) the denominator is 1 and hence

$$\square^* = \sum_{r=1}^{8} u_r^{¿} Yro \quad (1.10)$$

As mentioned earlier, ($v^{¿} u^{¿}$) are the set of most favorable weight for the DMUo in the since of maximizing the ratio scale.

v i is the optimum weight for the input item i and it's magnitude expressed how highly the item is evaluated , relatively speaking . Similarly, $u_r^{¿}$ does the same for the output item r .furthermore, if we examine each item $v_i^{¿} xio$ in the virtual input?

$$\sum_{i=1}^{m} v_i^{¿} xio. \quad =1 \qquad (1.11)$$

Then we can see the relative importance of each item by reference to the value of each $v_i^{¿} xio$, the same situation hold for $u_r^{¿} Yro$ where the $u_r^{¿}$ provide a measure of $\square^*$.

These value not only show which item contribute to the evaluation of DMUo , but also what extent they do so.

i. **The CCR model and dual problem:**

Based on the matrix (x . y) as linear program with row victor v for input multiplier and row vector u as output multiplier . These multipliers are treated as variable in the flowing (LP) problem:

(LPo) max uYo **(1.12)**

Subject to:

$v x_o$ = 1**(1.13)**

- $vx+¿$ u y ≤ 0 **(1.14)**

v ≥ 0 , u ≥ 0 **(1.15)**The dual problem of (LPo) is expressed with areal variable and non-negative vector = ($❑_1$, ..., $❑_n ¿^T$ of variables as follows:

(DLPo) min **(1.16)**

Subject to: x_o – x ≥ 0 **(1.17)**

y ≥ Y **(1.18)**

≥ 0 **(1.19)**

Correspondence between the primal (LPo) and dual (DLPo) constraint and variable are displayed in the coming table (1.1):

Table (3.2.1), Primal and dual correspondence

Constraint dual variable (LP o) (DLPo)	Constraint dual variable (DLPo) (LP o)
$Vx_0 = 0$ □ $- Vx + u y \leq 0$ □≥ 0	□V_0 - x □≥ 0 V ≥ 0 y□≥ Y_ou ≥ 0

(DLPo) has a feasible solution □ = 1 ,$□_o$ = 1, □ = 1 (j ≠ 0) .

Hence the optimum □ , denoted □* , is not greater than 1 , on the other hand , due to the non-zero (*i.e.,* semi positive) assumption for the data , the constraint (3.8) forces (□) to be non-zero because $Y_o \geq 0$ and $Y_o \neq 0$. Hence from (3.7) ,□ must be greater than zero .

Putting all this together , we have 0 <□≤ 1 .the constraint of (LPo) require the activity (□ x_o, Y_o) to belong P . While the objective seek the minimum □that reduce the inputvector $x_o$radialy to □ x_o while remaining P .

In (DLPo) we are looking for input that guarantees at least the output level Y_o of DMUo in all components while reducing the input vector x_o proportionally to a value as small as possible .

It can be said that (x□ ,y□) outperform (□ x_o, Y_o) when □< 1 .

With regard to this property, we define the input excesses $S^{-} \in R^m$ and output shortfall $S^{+} \in R^s$ and identify them as" aslack" vector by :

$S^{-} = \square\ x_{o} - X\square$, $S^{+} = Y\square - Y_{o}$ **(1.20)**

With $S^{-} \geq 0$, $S^{+} \geq 0$ for any possible solution (□ **,** □ **)** of(LPo) .

To discover the possible input excesses and output shortfall , we solve the following tow- phase LP problem.

ii. **Phase** I

We solve (DLPo) .let the optimum solution value be □ . by the dually theorem of linear program , □ is equal to optimum objective value of (LP o) and CCR – efficiency value , also called " Farrell efficiency", this value of □ is incorporate in the following phase two extension of (LPo).

iii. Phase II

Using our knowledge of □ , we solve the following LP using (□ , S^{-} , S^{+}) as variables :

$$\text{Max } \omega = eS^{-} + eS^{+} \qquad (1.21)$$

Subject to : $S^{-} = \square\ x_{o} - X\square$ (1.22)

$$S^{+} = Y\square - Y_{o} \qquad (1.23)$$

$$\square \geq 0 ,\ S^{-} \geq 0 ,\ S^{+} \geq 0 .$$

Where e = (1,…,1) (vector for ones) so that e $S^{-¿} = \sum_{i=1}^{m} ¿ S_i^{-¿}$ and $S^{+¿} = \sum_{r=1}^{s} ¿ S_r^{+¿}$.

The objective of phase II is to find asolution that maximize the some of input excesses and output shortfall while keeping = $❑^{¿}$.

We should note that we could replace the objective term (1.21) with any weight sum of input excesses and output shortfall such as:

$$\omega = \omega_x S^{-¿} + \omega_y S^{+¿} \qquad (1.24)$$

Where the weight ω_x , ω_y are positive row vector. The modified objective function may result in different optimum solution for phase II . However, we can have the optimum $\omega^{¿} \geq 0$ in(1.21) if and only non-zero value is also obtained when the objective in (1.21) is replaced with (1.24) .Thus the objective in (1.21) will identify some zero slack with inefficiency if and if some non-zero(possible deferent) slack are identify with in efficiency in (1.24).

Definition (1.2) (max slack , zero slack activity)

An optimum ($❑^{¿}$, $S^{-¿}$, $S^{+¿}$) of phase II is called max slack solution .if the max –slack solution satisfies $S^{-¿}$ = 0 and $S^{+¿}$ = 1, then it is called zero slack.

Definition (1.3):(CCR – efficiency, ratio efficiency, technical efficiency)

If an optimum solution (❑¿ , ❑¿ , $S^{-¿}$, $S^{+¿}$) of two LPs above satisfies ❑¿ = 1 and is zero slack ($S^{-¿}$ =1, $S^{+¿}$ =1) then the DMUo is called CCR- efficiency . Otherwise the DMUo is called CCR- in efficiency because (i) ❑¿ = 1 (ii) all slack are zero. Must both be satisfied if full efficiency is to be obtained?

The first of this two conditions are refer to as "radial efficiency" it is also refer to as" technical efficiency" because a value ❑¿ < 1 mean that all input can be simultaneously without altering (= proportion) in which they are utilized . Because(1 - ❑¿) is the maximal proportionate reducing allowed by the production possibility set ,any further reducing associate with non zero slack will necessarily change the input proportion. Hence the in efficiency associated with any non-zero slack identify in the above two phase reduce are refer to as (mix efficiency).

Other name is used to characterize two source of in efficiency. For instance the term ' weak efficiency " is also some time used when attention restricted (i) in definition **(** 3.2**)** the condition (i) and (ii) taken together describe what is called " Pareto –Koopmans" efficiency which can be verbalized as follow:

Definition(1.4) (**Pareto –Koopmans efficiency)**

A DMU is fully efficiency if and only if it is not possible to improve any input or output without worsening some of input or output.

The CCR –efficiency given in definition (3.2) is equivalent to that given by definition (2.1)

Proof. First, notice that the vector V and u of (LPo) and dual multipliers corresponding to the constraints(3.7) and (3.8) of (DLPo) respectively . See table (3.1) .Now the following "complementary condition" hold between any optimum solution ($v^{¿}$, u*) of (LPo) and (❑$^{¿}$, $S^{-¿}$, $S^{+¿}$) of (DLPo) .

This means that if any component of $v^{¿}$, $u^{¿}$ is positive then the corresponding of $S^{-¿}$ or $S^{+¿}$ must be zero , and conversely , with the possibility also allowed in which both component may be zero simultaneously .

Now we demonstrate definition (3.2) implies definition (2.1)

(i) If ❑$^{¿}$ <1 , then DMUo is inefficient by definition (2.1) since (LPo) and(DLPo) have the same optimum objective value ❑$^{¿}$.

(ii) If ❑$^{¿}$ = 1 and is not zero slack ($S^{-¿} \neq$ o and $S^{+¿} \neq$ 0) then by the complementary condition above, the element of $v^{¿}$, $u^{¿}$ corresponding to the positive slack must be non-zero.

Thus, the DMUo is in efficient by definition (2.1) .

(iii) Lastly if ❑$^{¿}$ = 1 and zero slack, then by the (strong theorem of complementary, (LPo) is assumed is assured of positive optimum solution ($v^{¿}$, $u^{¿}$) and hence DMUo is CCR-inefficient by definition (2.1) .

The reverce is also true by the complementary theorem between ($v^{¿}$, $u^{¿}$) and ($S^{-¿}$, $S^{+¿}$).

(iv)The reference set and improvement efficiency

Definition (1.5) (reference set)

For an inefficient DMUo , we define its reference set E_o , based on the max –slack solution as obtained in phases one and two by

E_o ={j ❑$^{¿}_{j}$ > 0} (j $\in$ { 1,…,n})

An optimum solution can be expressed as

$❑^{¿}$ x_o = $\sum_{j \in Eo} x_j ❑^{¿}_j$ + $S^{-¿}$. (1.24)

Y_o = $\sum_{j \in Eo} Y_j ❑^{¿}_j$ - $S^{+¿}$. (1.25)

This can be interpreted as follows:

$x_o \geq \theta^{¿}$ x_o - $S^{-¿}$ = $\sum_{j \in Eo} x_j ❑^{¿}_j$ (1.26)

This means xo ≥ technical – mix efficiency

= appositive combinations of observed input value ( )

Also $Y_o \leq Y_o$ + $S^{+¿}$ = $\sum_{j \in Eo} Y_j ❑^{¿}_j$ (1.27)

Means

$Y_o \leq$ observed output + shortfall

= appositive combination of observed output value ( )

These relation suggest that the efficiency of (x_o , Y_o) for DMUo can be improve if input value are reduced radially by $\theta^{¿}$ by ratio $\theta^{¿}$ and input excesses recorded in $S^{-¿}$ are eliminated.Similarly efficiency can be attained if the output values are augmented by the output shortfalls in $S^{+¿}$.

(v) Reason for solving CCR model using (DLPo)

It is not advisable to solve (LPo) directly . The reason is:

(i) The computational effort of LP apt to grow in proportion to power of the number constraints. Usually in DEA, n, the number of DMUs is considerable largely than (m +s) the number of input and output, and hence it take more time than to solve (DLPo) which (m+ s) constraints . In condition, since the memory size needed for keeping these basis (or its increase) is square of the number of constraints (DLPo) is better fitted for momery

(ii) We cannot the pertinent max-slack solution by (LPo) .

(iii) The interpretation (DLPo) are more straight word the solution are characterized as inputs and out puts that correspond to the original data whereas multipliers provided by solution of these observe value . These value are also important, ofcourse , but they are generally best reserved for supplementary analysis after solution to (DLPo) is achieved.

(vi) The output oriented models

The type of model that attempts to maximize output while using no more than the observed amount of any inputs is refer to as the output oriented model , formulated as:

(DLPo) max = η (1.28)

Subject to x_o - x μ≥ 0 (1.29)

ηY_o - Y μ≤ 0 (1.30)

μ≥ 0 (1.31)

An optimum solution of (DLPo) can be derived directly from an optimum solution of input oriented model as follow:

= μ/ , = 1 / η (1.32)

Then (DLPo) become

(DLPo) min (1.33)

Subject to x_o - x ≥0 (1.34)

Y_o - Y ≤ 0 (1.35)

≥ 0 (1.36)

This is input oriented CCR model. Thus, an optimum solution of the output oriented model related to that of the input oriented model ratio.

$\eta^{¿}$ = 1 / $\theta^{¿}$, $U^{¿}$ = ❑$^{¿}$ / $u^{¿}$.

We can conclude that an input oriented CCR model will be in efficient for any DMU if and only if it is also efficient when the output oriented CCR model is used to evaluate its performance.

3.2.2. The basic BCC model:

The BCC model evaluates the efficiency of DMUo (0= 1,…, n).

This model which developed by (banker, Charnes, and cooper, 1984) .below is short a brief of the two way of BCC approaches the input oriented and output oriented.

i. The input oriented

The inputs orientation BBC model evaluate the efficiency of the DMUs (o = 1, 2 ,… ,n1) by solving the following (envelopment form) linear program:

BCC min $\square_B$(2.1.)

Subject to $\square_B$ x_o - x$\square$≥ o (2.2)

y$\square$≥ y_o(2.3)

$\square$≥o(2.4)

Where $\square_B$ isscalar.

The dual multiplier from this linear program (BCC) is expressed as:

Max z = u ^{Y} $_o$ - u_o(2.5)

Subject to $vx_o = 1$ (2.6)

vx + u y –u $_o$ e ≤ 0 (2.7)

V≥ 0, u≤ 0, u_o free sign

Where z and u are scalar and the latter being "free sign" may be positive or negative (or zero). The equivalent BCC fractional program is obtained from the dual program as:

Max $¿\frac{u_y - u_0}{u_{xj}}$ = (2.8)

Subject to $\frac{u_{yj} - u_0}{v_{xj}}$ (2.9)

v≥ 0 , u ≥ o , u $_o$ free (2.10)

Table (3.2.2) primal dual correspondence in BCC model.

Envelopment form multiplier form Constraint variable	Envelopment form multiplier form Constraint variable
B xo –x ^{Y} ≥0 V ≥ 0 Y≥ ^{Y} $_o$ μ≥ 0	V x_0= 1 - V x + u y – u_o e ≤ 0 ≤ 0

e□ = 1 u o	

Correspondence between the primal dual constraints and variable can be diagramed ad in above table.

It is clear that difference between CCR and BCC is present in the free variable μ_{0}, which is dual variable associate with the constraint e□ = 1 which also does not appear in CCR model.

The primal problem (BCC) is solving using two phase procedure similar to CCR case. in the first phase , we minimize $□_B$ and , in the second phase , we maximize the sum input excess and output shortfall , keeping $□_B$ = $□^{¿}_{B}$ (optimum objective value) . The evolution secured from CCR and BCC model are also related to each other as follow. Let an optimum solution for BCC be ($□^{¿}_{B}□^{¿}$, $S^{-¿}$, $S^{+¿}$), where $S^{-¿}$ and $S^{+¿}$ represent the maximal input excess and output shortfall, respectively. Notice that $□^{¿}_{B}$ is not less than the optimum objective value $□^{¿}$ of CCR model, since BCC impose one additional constraint, e □ = 1, so its feasible region is subset of that of the CCR model.

Basing on the above model for each DMU , BCC- efficiency score are obtained (with similar interpretation of its value as in the CCR model) , these score are also called " pure technical efficiency score " , since they are obtained from the model that allows variable return to scale and hence eliminate the " scale part" of the efficiency from the analysis . Generally, for each DMU the CCR-efficiency score will not exceed the BCC-efficiency score, what is intuitively clear since in the BCC- model each DMU is analyzed "locally" rather than globally.

Definition (2.1) (BCC – efficiency):

If an optimum solution ($\square^{¿}_{B}$, $\square^{¿}$, $S^{-¿}$, $S^{+¿}$) for BCC satisfies $\square^{¿}_{B}$ = 1 and has no slack ($S^{-¿}$ = 0 , $S^{+¿}$ = 0) , then the DMUo is called BCC –efficient , otherwise it is BCC inefficient.

Definition (2.2) (reference set):

For the BCC – efficient DMUo, we define its reference set $E_{0,}$ based on optimum solution $\square^{¿}$ by

E_0 = {j□ $\square^{¿}{}_{j}$> 0} (j $\in$ {1,..., n }) (2.11)

If there are multiple optimum solutions, we can choose any one to find that

$$\square^{¿}_{B}\ x_o = \sum_{j\in E_o} \square^{¿}{}_{j}\ x_j + S^{-¿} \qquad (2.12)$$

$$Y_o = \sum_{j\in E_o} \square^{¿}{}_{j}\ y_j - S^{+¿} \qquad (2.13)$$

Thus, we have a formula for improving, the BCC schemes ion

$$\acute{x}_o + \Leftarrow \square^{¿}{}_{B} x_o - S^{-¿} \quad (2.14)$$

$$\acute{y}_o + \Leftarrow Y_o - S^{+¿} \quad (2.15)$$

To give more idea of DMUs efficiency according to (DEA) method, figure (3.2.2) ,exhibit 4 DMUs A,B,C ,and D each with one input and one output .

Figure (3.2.2) , 4 DMUs A,B,C ,and D each with one input and one output .

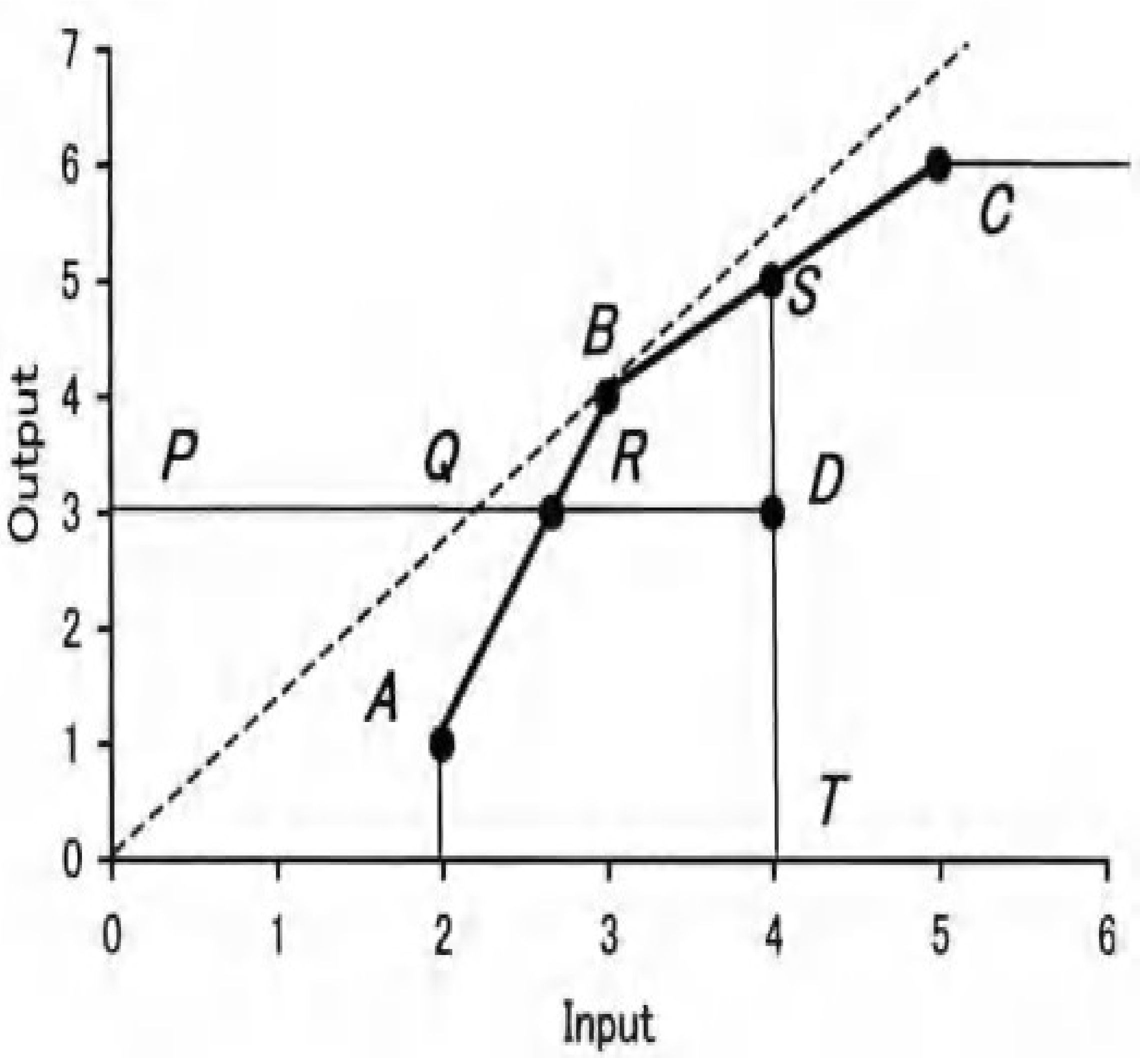

Figure (3.2.2) exhibit 4 DMUs A, B, C, and D each with one input and one output.

The efficient frontier CCR model consists of bold line connecting A, B, and C. the production possibility set is the area consisting the frontier together with observed or possibility activities with an excess of input and / or shortfall in output compare with frontier A, B, and D are on the frontier and the BCC efficient however, only B is CCR – efficient.

ii. The output –oriented BCC model

The output –oriented BCC model evaluate the efficiency of the DMUs (o = 1, 2 ,… ,n) by solving the following (envelopment form) linear program:

(BCCCOo) max ηB (2.16)

Subject to x ≤ x_o (2.17)

$\eta B Y_o$ – Y ≤ o (2.18)

≥ 0 (2.19)

This is envelopment from the output oriented BCC model. The dual (multiplier) form associated with the above linear program (BCCCo) is expressed as:

Min z = $v x_o$ - v o (2.20)

Subject to u Y_o = 1 (2.21)

V x – u Y – V o e ≥ 0 (2.22)

V ≤0, u ≥ 0 , V o free sign   (2.23)

Where V o is scalar associated with e   = 1 in the envelopment model. Finally we have the equivalent (BCC) fractional programming for the latter (multiplier) model:

Min $\frac{vx_o - V_o}{uY_o}$ (2.24)

Subject to $\frac{vx_i - V_o}{uY_i}$ ≥1 (j = 1… n) (2.25)

V≥0, u ≥ 0 , Vo free sign .

Employing a CCR model in envelopment form to obtain an optimum solution ($\square^{¿}_{1},\ldots,\square^{¿}_{n}$) , return to scale at this point can be obtained from the conditions:

(i) If $\sum_{j=1}^{n} \square^{¿}_{1}$ = 1 in any alternate optimum then constant –return to scale prevail.

(ii) If $\sum_{j=1}^{n} \square^{¿}_{1}$ > 1 for any alternate optima, then decreasing return to –scale prevail.

(iii) If $\sum_{j=1}^{n} \square^{¿}_{1}$ < 1 for all alternate optima then increasing return to scale prevail.

3.2.3 TheMalmquist (TFP) index:

The Malmquist index is the chain index approach, which measures a change in efficiency relative to a base year.

This approach measures change in efficiency with respect to a base year value of 1 , if the index for the year other than the base year, is above 1 , there is an efficiency improvement , if the index value for the year is below 1 , there is efficiency regress . This change in efficiency can be decomposed into components due to change in the technical efficiency and movement due to change in technology.

Change in firm's technical efficiency can be decomposed into change due to pure technical efficiency change and due to scale efficiency.

This measure of productivity change accommodates for multi –output and multi-input production processes and does not require the definition of specific functional form for the production technology.

The Malmquist index has three main advantages according to the Fischer and Tornqvist indices.

Firstly, it does not require the profit maximization, or the cost minimization, assumption.

Secondly, it does not require information on the input and output prices.

Finally, if the researcher has panel data, it allows the decomposition of productivity changes into two components (technical efficiency change or catching up, and technical change or changes in the best practice).

The main disadvantage of using Malmquist index can represent its necessity to compute the distance functions. However, the Data Envelopment Analysis (DEA) technique can be used to solve this problem.

IT is necessary to explain what is an index is . indexes are the tools that are used to measure the change in the level of economical variable , an index number is define as a real number which measure the change in set related variables. They are used to compares the value of variables that change by time.

MalmquistFactorindex (TFI) measure the (TFP) change between two data point by calculating ratio of the distance of each data point relative to common technology. The distance function is used for this measurement. Distance function defines the production technology for multiple outputs and multiple inputs without any need for cost minimization of profit maximization objectives.

Input distance function define the production technology according to the most contracted input vector when the output vector is given .similarly, output distance define the production technology according to the most expanded input vector when the input vector is given.

The input distance function for firm i with respect to two time period, t and S, is defined using following Equation (3.1) where $S^t = \{(x^t, y^t): x^t \Rightarrow y^t\}$ is the production technology thatgoverns the transformation of inputs for period t :

$$d_o^t(x^s, y^s) = \min\{\theta > 0 : (y^s, \theta x^s) \in S^t\} \quad (3.1)$$

The distance function in Equation (2) measures the minimum proportional change in input usage at period s required to make the period s input-output set, (x^s, y^s), feasible in relation to the technology St at period t. the Malmquist productivity index comparing periods t and t +1 can then be defined using distance functions representing the four combination of adjacent time periods

This study adopts the output-oriented Malmquist productivity change index, referring the emphasis on the equi-proportionate increase of outputs, within the context of a given level of input. The output-oriented Malmquist productivity change index.

According to Fare (1994) the input oriented Malmquist(TPF) change index between time period Sand t , this can be expressed as follows:

$$m_o = (\acute{x}_s, \acute{y}_s, \acute{x}_t, \acute{y}_t) = \sqrt{\frac{\square_o^{ds}(\vec{x}_t, \vec{y}_t) * \square_o^{dt}(\vec{x}_t, \vec{y}_t)}{\square_o^{ds}(\vec{x}_s, \vec{x}_s) * \square_o^{dt}(\vec{x}_s, \vec{y}_s)}} \quad (3.2)$$

Where: $\square_o^{ds}(\vec{x}_t, \vec{y}_t)$ indicate of the observation time S from the technology of time t. If the function m (3 .2) > 1 then it means that TPF increase from time to time t. In opposite, if the function m (3. 2) < 1 then it means that TPF decreases from time to time t. Following Fare et al., (1994) an equivalentway of writing Equation (3) is

$$m_o = (\acute{x}_s, \acute{y}_s, \acute{x}_t, \acute{y}_t) = \frac{\square^{dt}_{o}(\vec{x}_t, \vec{y}_t)}{\square^{ds}_{o}(\vec{x}_s, \vec{x}_s)} \sqrt{\frac{\square^{ds}_{o}(\vec{x}_t, \vec{y}_t) * \square^{ds}_{o}(\vec{x}_s, \vec{y}_s)}{\square^{dt}_{o}(\vec{x}_t, \vec{x}_t) * \square^{dt}_{o}(\vec{x}_s, \vec{y}_s)}} \quad (3.3$$

The fist term on the right hand side of the equation is the measure of Farrell's output oriented efficiency change between time S and time t. Additionally, the term in the square root determine the technical change.

Catch-up or recovery is related to the degree in which a decision making unit (DMU) improves or worsens efficiency frontier shift (or innovation) is a term which reflects the change in the efficiency its frontiers between the two time periods (Berket&Lalitha 2012).

In addition, technical efficiency change can be further decomposed into pure technical efficiency change and scale efficiency change.

Therefore, the two terms in equation (1) are:

Efficiency change:

$$m_o = (\acute{x}_s, \acute{y}_s, \acute{x}_t, \acute{y}_t) = \frac{\square^{dt}_{o}(\vec{x}_t, \vec{y}_t)}{\square^{ds}_{o}(\vec{x}_s, \vec{x}_s)} \quad (3.4)$$

Technical change:

$$m_o = (\acute{x}_s, \acute{y}_s, \acute{x}_t, \acute{y}_t) = \sqrt{\frac{\square^{ds}_{o}(\vec{x}_t, \vec{y}_t) * \square^{ds}_{o}(\vec{x}_s, \vec{y}_s)}{\square^{dt}_{o}(\vec{x}_t, \vec{x}_t) * \square^{dt}_{o}(\vec{x}_s, \vec{y}_s)}} \quad (3.5)$$

Malmquist (TFP) index can be evaluated by two different approaches which are the parametric and non-parametric.

In parametric approach the distance functions are determined by parametric method, in the other words, the production frontier stochastic frontier, however in non-parametric approach the distance functions are determined by non-parametric approach.

The Malmquist productivity index can be interpreted as a measure of total factors productivity (TFP) growth. Improvement in productivity, as well as improvement in efficiency and technology, is indicated by values greater than one, whereas value less than one indicate regress.

A. Technical efficiency:

Technical efficiency is expressed as the non-existence of any waste. In other words, technical efficiency is the success of producing the maximum output through utilizing the input in a most efficient way. It is a fact that all technical DMUs are located on the efficient frontier and the DMUs below the efficient frontier waste their resources relatively.

Given the fact that DEA is non- stochastic model, it's particularly sensitive to the problem of miss measurement, there for inclusion of those schemes in to the sample could seriously undermine the quality of the results.

It is important to note that scale efficiency can be affected by poor management within organization or disadvantageous operating environment. Thus, scale efficiency which is $\pi_i = \square_i / \sigma$ means the extent to which the scheme can take advantage of return to scale by altering its size towards optimum scale. The fraction of output lost due to scale inefficiency can be computed as (1- π_i). Scale efficiency equal one unit at any time along the CCR frontier line, at which production technology exhibit constant return to scale. Scale efficiency can arise due to variable (increasing or decreasing) return to scale. On the other hand, pure technical inefficiency occurs because a DMU uses more inputs than needed (input waste). Alternatively pure technical inefficiency can be caused by inefficient implementation of the production plan in converting input to output (managerial inefficiency) . However, scale inefficiency could be due to divergence of DMU from the most productive size. There for, decomposing technical efficiency into pure technical and scale efficiencies allows us to gain insight into the main source of inefficiency.

The coming chapter reviews descriptive side of the study ;Agricultural sector of the Sudan it gives light on The Sudanese mechanized and rain fed schemes in south Kordofan and Gedaref state. Also the chapter high lights some factors that have direct effect on agricultural production efficiency such as geographical Features (soil type, climate characteristics) andagricultural policies.